$\boxed{A1}$ a) $\frac{1}{2-x} = \frac{1}{2(1-\frac{x}{2})} = \frac{1}{2}\,\frac{1}{(1-\frac{x}{2})} = f(x)$ 2

$f(x) = \frac{1}{2}\left\{1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \dots\right\} = \frac{1}{2}\sum_{k=0}^{\infty}\left(\frac{x}{2}\right)^k = \frac{1}{2}\sum_{k=0}^{\infty}\frac{x^k}{2^k}$ 1 1

geom. Reihe konvergent für $\left|\frac{x}{2}\right| < 1 \Rightarrow |x| < 2$ 1

$\Leftrightarrow -2 < x < 2 \Leftrightarrow r = 2 =$ Konv.radius 1

b) $g(x) = \frac{\sin(2x)}{2-x} = \frac{1}{2-x}\cdot\sin(2x)$ 1

$\sin(2x) = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} \pm \dots$ 1

$g(x) = \frac{1}{2}\left\{1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \frac{x^4}{16}\right\}\cdot\left\{2x - \frac{8x^3}{6} + \dots\right\}$ 2

$g(x) = \frac{1}{2}\left\{2x + x^2 + x^3\left(\frac{1}{2} - \frac{4}{3}\right) + x^4\left(\frac{1}{4} - \frac{2}{3}\right) + \dots\right\}$ 2

$\boxed{g(x) = x + \frac{x^2}{2} - \frac{5}{16}x^3 - \frac{5}{24}x^4 + \dots}$ 1

c) $\int_0^1 g(x)\,dx = \left[\frac{x^2}{2} + \frac{x^3}{6} - \frac{5}{48}x^4 - \frac{5}{120}x^5\right]_0^1 = \frac{1}{2} + \frac{1}{6} - \frac{5}{48} - \frac{1}{24} = \frac{25}{48}$

3 1 1

$\int_0^3 g(x)\,dx$ ex. nicht, da $3 > r = 2$ 2

A4

a) $V = V_E \tanh\left(\frac{g}{V_E} t\right)$

1

$$\Rightarrow s = \int v(t)\,dt = \int V_E \frac{\sinh\left(\frac{g}{V_E} t\right)}{\cosh\left(\frac{g}{V_E} t\right)}\,dt =$$

$$= \frac{V_E^2}{g} \ln\left(\cosh\left(\frac{g}{V_E} t\right)\right) + C$$ 2

1

$$s(0) \overset{!}{=} 0 \wedge s(0) = C \Rightarrow \boxed{C = 0} \Rightarrow \boxed{s(t) = \frac{V_E^2}{g} \ln\left(\cosh\left(\frac{g}{V_E} t\right)\right)}$$

1

b) $$a(t) = -\omega^2 \cos(\omega t) \Rightarrow v(t) = v = -\omega^2 \int \cos(\omega t)\,dt =$$

$$= -\frac{\omega^2}{\omega} \sin(\omega t) + C = -\omega \sin(\omega t) + C$$ 1

$$v(0) \overset{!}{=} 0 \wedge v(0) = C \Rightarrow \boxed{C = 0} \Rightarrow \boxed{v(t) = -\omega \sin(\omega t)}$$

1 1

$$s(t) = -\omega \int \sin(\omega t)\,dt = \boxed{\cos(\omega t) + C}$$ 1

$$s(0) \overset{!}{=} 1 \wedge s(0) = 1 + C \Rightarrow \boxed{C = 0} \Rightarrow \boxed{s(t) = \cos(\omega t)}$$

1 1

$\boxed{4c)}$ geometrische Folge

$h_n := h(n)$ ... Höhe nach n Schritten in cm.

$\Rightarrow$ $h(1) = 50$

$h(2) = 42{,}5$ 1

$h(3) = 36{,}13$

$\vdots$

geom. Folge mit $\boxed{q = 0{,}85}$ 1

$$\boxed{s_n = h_1 \cdot \frac{q^n - 1}{q - 1}}$$ 1 Summenformel einer geom. Reihe

Es gilt: $s_n = 50 \cdot \frac{0{,}85^n - 1}{0{,}85 - 1}$ 1

1

$$\Rightarrow s_\infty := \lim_{n\to\infty} s_n = \lim_{n\to\infty} 50 \cdot \frac{0{,}85^n - 1}{0{,}85 - 1} = \boxed{333{,}\overline{3}}$$ (cm)

1

d) Ist p_n der Preis (in €) einer n-zeiligen Anzeige, so gilt $\boxed{p_n = a + d \cdot n}$ (arithm. Folge) 1

$p_3 = a + 3d$ und $p_{12} = a + 12 \cdot d$

$\Rightarrow$ $\boxed{a = 46}$, $\boxed{d = 6}$ 1

$\Rightarrow$ $\boxed{p_n = 46 + 6n}$ 1

A5

a) <u>arithm. Folge</u>

$a_n = 600 \frac{km}{h} = 166{,}7 \frac{m}{s}$; $d = 9{,}8 \frac{m}{s}$ 1

$a_1 = 0 \frac{m}{s} \Rightarrow \quad a_n = a_1 + (n-1) \cdot d$

$\Leftrightarrow n = \frac{a_n - a_1}{d} + 1$ 1

$\Leftrightarrow n = \frac{166{,}7 - 0}{9{,}8} + 1 = 17{,}01$

$\Rightarrow \boxed{n \approx 17}$ 1

b) Für 12 Stunden gilt: $a_1 = 1$, $a_n = 12$, $n = 12$ 1

$\Rightarrow S_{12} = \frac{12}{2}(a_1 + a_n) = \frac{12}{2}(1 + 12) = 78$ 1

$\Rightarrow$ In ~~8~~ 5 Tagen macht die Turmuhr 10 ~~8~~ $\cdot 78 =$ ~~890~~ 780 Schläge 1

c) $a_1 = 49$, $a_n = \frac{1}{49}$, $n = 5$ ~~arithm.~~ geom. Folge

$a_n = a_1 \cdot q^{n-1} \Rightarrow q^{n-1} = \frac{a_n}{a_1} \Rightarrow q = \sqrt[n-1]{\frac{a_n}{a_1}}$ 1

$\Rightarrow q = \sqrt[4]{\frac{1}{49 \cdot 49}} \Rightarrow \boxed{q = \frac{1}{7}}$ 1

<u>ges.</u> Reihe lautet: $\boxed{49, 7, 1, \frac{1}{7}, \frac{1}{49}}$

1

$\boxed{A5}$ d) $0{,}\overline{34}$

$$a_1 = \frac{34}{100}, \quad a_2 = \frac{34}{10000}$$

$$0{,}\overline{34} = 0{,}343434\ldots \qquad \text{(geom. Reihe)}$$

$$= \frac{34}{100} + \frac{34}{10000} + \frac{34}{1000000} + \ldots \qquad 1$$

$$q = \frac{a_2}{a_1} = \frac{1}{100} < 1 \qquad 1$$

$$\Rightarrow \quad S_\infty = \frac{\frac{34}{100}}{1 - \frac{1}{100}} = \boxed{\frac{34}{99}} = 0{,}34\ldots \qquad 1$$

$\boxed{A5}$

e)

(i) $$\lim_{n\to\infty} \frac{27^{\log_3 n}}{16^{\log_2 n}} = \lim_{n\to\infty} \frac{3^{3\log_3 n}}{2^{4\log_2 n}}$$ 1

$$= \lim_{n\to\infty} \frac{n^3}{n^4} = \lim_{n\to\infty} \frac{1}{n} = 0$$

1 1

(ii) $$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{2n} \sqrt[n]{n+1}$$

1

$$= \left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right]^2 \cdot \lim_{n\to\infty} \sqrt[n]{n+1}$$

1

$$= e^2 \cdot 1 = \boxed{e^2}$$

1