$\boxed{A1}$

a) $z = 2 - j2\sqrt{3} = 4\,e^{-j\frac{\pi}{3}} = 4\angle -60^\circ$

b) $\left|\dfrac{3+5j}{1+4j}\right| = \left|\dfrac{(3+5j)(1-4j)}{(1+4j)(1-4j)}\right| = \sqrt{2}$

c) $z^3 = \dfrac{3\sqrt{2}}{2}(1+j) = 3e^{j\frac{\pi}{4}}$

$\Rightarrow \quad z_k = \sqrt[3]{3}\; e^{j\frac{(\frac{\pi}{4}+2k\pi)}{3}} \qquad (k = 0,1,2)$

$\Rightarrow \quad z_1 = \sqrt[3]{3}\; e^{j\frac{\pi}{12}}\; ; \quad z_2 = \sqrt[3]{3}\; e^{j\frac{9}{12}\pi}\; ; \quad z_3 = \sqrt[3]{3}\, e^{j\frac{17}{12}\pi}$

d) $4\cancel{A}\cos(\omega t) \longleftrightarrow z_{10} = \cancel{A}\,4$

$2\cos(\omega t) \longleftrightarrow z_{20} = 2e^{j\frac{2}{3}\pi}$

$A\cos(\omega t + \varphi) \longleftrightarrow z_0 = A e^{j\varphi}$

$\Rightarrow z_{10} + z_{20} = z_0 \Rightarrow 4 + 2e^{j\frac{2}{3}\pi} = Ae^{j\varphi}$

$\Rightarrow 4 + 2\left(\cos\left(\tfrac{2}{3}\pi\right) + j\sin\left(\tfrac{2}{3}\pi\right)\right) = \cancel{A}\; Ae^{j\varphi}$

$\Rightarrow 4 + 2\left(-\tfrac{1}{2} + j\,\tfrac{\sqrt{3}}{2}\right) = \cancel{A}\; Ae^{j\varphi}$

$\Rightarrow 4 - 1 + j\sqrt{3} = Ae^{j\varphi} \Longleftrightarrow 3 + \sqrt{3}j = Ae^{j\varphi}$

$\Rightarrow \quad \boxed{A = \sqrt{12} = 2\sqrt{3}} \qquad \boxed{\varphi = \arctan\left(\tfrac{\sqrt{3}}{3}\right) = \tfrac{\pi}{6} \mathrel{\hat{=}} 30^\circ}$

$\boxed{A2}$

a) $z = \dfrac{e^{j\frac{\pi}{3}}}{(1+j)^2} = \dfrac{\frac{1}{2}+\frac{1}{2}\sqrt{3}j}{2j}\cdot\dfrac{j}{j} = \frac{1}{4}\left(\sqrt{3}-j\right)$

$\Longrightarrow \boxed{\operatorname{Im}(z) = -\frac{1}{4}}$

b) $3z_1 - \dfrac{5}{z_2} = 6+3j-5\cdot\dfrac{1}{(1-2j)}\cdot\dfrac{(1+2j)}{(1+2j)} = 6+3j-1-2j = 5+j$

$\Longrightarrow |5+j| = \sqrt{(5+j)(5-j)} = \sqrt{26}$

c) $z^4 = 2\left(-1-\sqrt{3}j\right) = 2\cdot 2\cdot e^{j\left(-\frac{2\pi}{3}+2k\pi\right)}$

$\Longrightarrow z_k = \sqrt{2}\,e^{j\left(-\frac{\pi}{6}+k\frac{\pi}{2}\right)} \quad (k=0,1,2,3)$

1. Quadrant : $k=1 \Longrightarrow z_1 = \sqrt{2}\,e^{j\frac{\pi}{3}} = \sqrt{2}\left(\frac{1}{2}+j\frac{1}{2}\sqrt{3}\right)$

$\Longrightarrow \boxed{\operatorname{Re} z_1 = \frac{1}{2}\sqrt{2}} \qquad \boxed{\operatorname{Im} z_1 = \frac{1}{2}\sqrt{6}}$

d) $|z-1| = 2|z+1| \Longrightarrow \sqrt{(x-1)^2+y^2} = 2\sqrt{(x+1)^2+y^2}$

$\Longleftrightarrow 3x^2+10x+3+3y^2 = 0$

$\Longleftrightarrow 3\left(x^2+2\cdot\frac{5}{3}x+\frac{25}{9}\right)+3y^2 = -3+\frac{25}{3} = \frac{16}{3}$

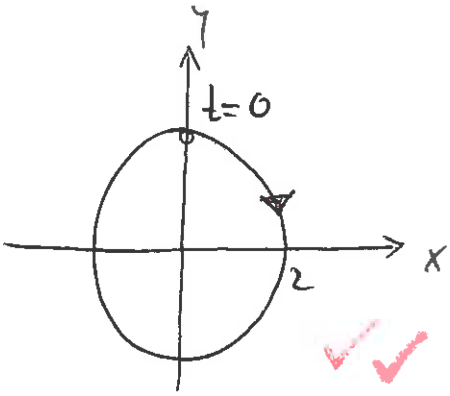
$\Longleftrightarrow \left(x+\frac{5}{3}\right)^2+y^2 = \left(\frac{4}{3}\right)^2$

Kreis um $\left(-\frac{5}{3}\middle|0\right)$; $r = \frac{4}{3}$

e) $$z(t) = x(t) + y(t) = \frac{2j}{\cos t + j\sin t} = 2j\,\frac{\cos t - j\sin t}{\cos^2 t + \sin^2 t} =$$

$$= 2\sin t + 2j\cos t$$

$$\Rightarrow \quad |z| = 2 \iff x^2 + y^2 = 4$$



$\boxed{A3}$

a) $\displaystyle\int \frac{e^{ax}}{1+e^{ax}}\,dx \underset{z:=e^{ax}}{=\!=} \frac{1}{a}\int \frac{dx}{1+z}\,dz =$

$z:=e^{ax}$

$\frac{dz}{dx} = ae^{ax}$

$dx = \frac{dz}{ae^{ax}}$

$= \frac{1}{a}\ln|1+z| + C \underset{R.S}{=} \frac{1}{a}\ln|1+e^{ax}| + C$

b) $\displaystyle\int \frac{x}{\sqrt{(x^2+25)^3}}\,dx \underset{z:=x^2+25}{=\!=} \frac{1}{2}\int\frac{dz}{\sqrt{z^3}} = \frac{1}{2}\int z^{-\frac{3}{2}}\,dz =$

$z:=x^2+25$

$\frac{dz}{dx} = 2x$

$dx = \frac{dz}{2x}$

$= \frac{1}{2}\cdot z^{-\frac{1}{2}}(-2) + C \underset{R.S}{=} -\frac{1}{\sqrt{x^2+25}} + C$

c) $\displaystyle\int \frac{\sin^3 x}{\cos x}\,dx \underset{z:=\sin x}{=\!=} \int\frac{z^3}{1-z^2}\,dz = \int\left(-z+\frac{z}{1-z^2}\right)dz$

$z:=\sin x$

$\frac{dz}{dx} = \cos x$

$dx = \frac{dz}{\cos x}$

$z^3 : (1-z^2) = -z + \frac{z}{1-z^2}$

$\underline{-(z^3-z)}$

z

$= \int\left(-z - \frac{1}{2}\frac{2z}{1-z^2}\right)dz$

$= -\frac{1}{2}z^2 - \frac{1}{2}\ln|1-z^2| + C$

$\underset{R.S.}{=} -\frac{1}{2}\sin^2 x - \frac{1}{2}\ln|1-\sin^2 x| + C$

$= -\frac{1}{2}\sin^2 x - \frac{1}{2}\ln|\cos^2 x| + C$

$= -\frac{1}{2}\sin^2 x - \ln|\cos x| + C$

$\boxed{A4}$

a) $\lim\limits_{x\to 1} \dfrac{x-1-\ln x}{(x-1)^2} \underset{L'H.}{=} \lim\limits_{x\to 1} \dfrac{1-\frac{1}{x}}{2(x-1)} \underset{L'H.}{=} \lim\limits_{x\to 1} \dfrac{\frac{1}{x^2}}{2} = \underline{\underline{\dfrac{1}{2}}}$

b) $\lim\limits_{x\to 0} \dfrac{x\sin(2x)}{e^x-2+e^{-x}} \underset{L'H.}{=} \lim\limits_{x\to 0} \dfrac{\sin(2x)+2x\cos(2x)}{e^x-e^{-x}} \underset{L'H.}{=}$

$\lim\limits_{x\to 0} \dfrac{2\overset{\to 1}{\cos(2x)} + 2\overset{\to 1}{\cos(x)} - \overset{\to 0}{4x\sin(2x)}}{e^x+e^{-x}} = \dfrac{4}{2} = \underline{\underline{2}}$

$\boxed{A6}$

a) $y = y(u(v(w(x))))$

$w(x) = x^2+1 \Rightarrow \dfrac{dw}{dx} = 2x$

$v(w) = \cos w \Rightarrow \dfrac{dv}{dw} = -\sin w$

$u(v) = v^2 \Rightarrow \dfrac{du}{dv} = 2v$

$y(u) = \ln u \Rightarrow \dfrac{dy}{du} = \dfrac{1}{u}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dv}\cdot\dfrac{dv}{dw}\cdot\dfrac{dw}{dx}$

$= \dfrac{1}{u}\cdot 2v\,(-\sin(w))\cdot 2x = -\dfrac{1}{v^2}\cdot 2v\sin(x^2+1)\cdot 2x =$

$\cancel{= \dfrac{1}{4x^2}\cdot 2\cos(w)\sin(x^2+1)\cdot 2x = -\dfrac{1}{x}}$

$$= -\frac{1}{\cos^2 w} \cdot 2\cos w \cdot \sin(x^2+1) \cdot 2x$$

$$= -\frac{4x}{\cos(x^2+1)} \cdot \sin(x^2+1) = \boxed{-4x\tan(x^2+1)}$$

b) $\int_1^\infty x e^{ax}\,dx$

Betrachte: $\int_1^u \underbrace{x}_{v}\underbrace{e^{ax}}_{u'}dx = \left[\frac{1}{a}e^{ax}x\right]_1^u - \frac{1}{a}\int_1^u e^{ax}\,dx =$

$$= \frac{1}{a}ue^{au} - \frac{1}{a}e^{a} - \frac{1}{a^2}\left[e^{ax}\right]_1^u =$$

$$= \frac{1}{a}ue^{au} - \frac{1}{a}e^{a} - \frac{1}{a^2}e^{au} + \frac{1}{a^2}e^{a}$$

$$I(a) := \int_1^\infty xe^{ax}dx = \lim_{u\to\infty}\int_1^u xe^{ax}$$

Int. ex falls $\boxed{a<0}$ Dann

$$\lim_{u\to\infty}\left(\frac{1}{a}\underbrace{ue^{au}}_{\to 0} - \frac{1}{a}e^{a} - \frac{1}{a^2}\underbrace{e^{au}}_{\to 0} + \frac{1}{a^2}e^{a}\right) =$$

$$= e^{a}\left(\frac{1}{a^2} - \frac{1}{a}\right) = e^{a}\,\frac{1-a}{a^2}$$

$\boxed{A5}$

$$\frac{1-\cos x}{x^2} = \frac{1-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+-\dots\right)}{x^2} =$$

$$= \frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \frac{x^6}{8!} -+\dots \qquad (x \in \mathbb{R})$$

$$\Rightarrow \int_0^1 \frac{1-\cos x}{x^2}\,dx \approx \int_0^1 \left(\frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \frac{x^6}{8!}\right) dx$$

$$= \frac{x}{2!} - \frac{x^3}{3\cdot 4!} + \frac{x^5}{5\cdot 6!} - \frac{x^7}{7\cdot 8!} \Bigg|_0^1$$

$$= \frac{1}{2!} - \frac{1}{3\cdot 4!} + \frac{1}{5\cdot 6!} - \frac{1}{7\cdot 8!}$$

$$= 0{,}5 - 0{,}01389 + 0{,}00028 \approx 0{,}4864$$

$$|\text{Fehler}| = \frac{1}{7\cdot 8!} \approx 3{,}54\cdot 10^{-6} < \frac{1}{1000}$$

$\boxed{A7}$ $y'' + 2y' + (1+\omega^2)y = 0$

a) $\Rightarrow$ <u>char. P.</u> $\lambda^2 + 2\lambda + (1+\omega^2) = 0$

$$\Rightarrow \lambda_{1/2} = \frac{-2 \pm \sqrt{4-4(1+\omega^2)}}{2} = -1 \pm \sqrt{-\omega^2} = -1 \pm j\omega$$

$$\Rightarrow y_h = e^{-x}\left(C_1 \cos(\omega x) + C_2 \sin(\omega x)\right) \quad \text{für } \omega \neq 0$$

$$y_h = e^{-x}(C_1 + C_2 x) \quad \text{für } \omega = 0$$

b) $\omega = 0$: <u>Ansatz</u>

(1) $y = Ax^2 e^{-x}$

(2) $y' = 2Axe^{-x} - Ax^2 e^{-x}$

(3) $y'' = 2Ae^{-x} - 2Axe^{-x} - 2Axe^{-x} + Ax^2 e^{-x}$

(1), (2), (3) in $y'' + 2y' + y = e^{-x}$:

$$2Ae^{-x} - 2Axe^{-x} - 2Axe^{-x} + Ax^2 e^{-x} + 4Axe^{-x} - 2Ax^2 e^{-x} + Ax^2 e^{-x} = e^{-x}$$

$$\Rightarrow 2Ae^{-x} = e^{-x} \Rightarrow \boxed{A = \frac{1}{2}}$$

$$\Rightarrow y_{allg} = y_h + y_p = C_1 e^{-x} + C_2 x e^{-x} + \frac{1}{2} x^2 e^{-x}$$