DGL des logistischen Wachstums

DGL: $B'(t) = kB(t)(S - kB(t))$; $B(0) = B_0$ **AWP**

kurz: $B' = kB(S - B) = kSB - kB^2$ (*)

Bernoulli-DGL: : (1) $B = \frac{1}{u} \Rightarrow B' = -\frac{1}{u^2}u'$ (2)

$$\underset{(1),(2)\text{ in }(*)}{\Longrightarrow} -\frac{1}{u^2}u' = kS\cdot\frac{1}{u} - \frac{k}{u^2} \quad \Big| \,(-u^2)$$

$$\Rightarrow u' = -kSu + k$$

$$\Rightarrow u' = k(1 - Su) \Rightarrow \int \frac{du}{1-Su} = k\int dt$$

$$\Rightarrow -\frac{1}{S}\ln|1 - Su| = kt + C_1 \quad \Big| \cdot(-S)$$

$$\Rightarrow \ln|1 - Su| = -kSt \underbrace{- SC_1}_{=:C_2} = -kSt + C_2$$

$$\Rightarrow |1 - Su| = e^{-kt}\cdot e^{C_2}$$

$$\Rightarrow 1 - Su = \underbrace{\pm e^{C_2}}_{=:C_3}\cdot e^{-kSt} = C_3\cdot e^{-kSt}$$

$$\Rightarrow Su = 1 - C_3 e^{-kSt} \quad \Big| : S$$

$$\Rightarrow u = \frac{1}{S} - \frac{C_3}{S}e^{-kSt}$$

$$\underset{RS}{\Longrightarrow} \frac{1}{B} = \frac{1}{S} - \frac{C_3}{S}e^{-kSt}$$

$$\Rightarrow B = \frac{1}{\frac{1}{S} - \frac{C_3}{S}e^{-kSt}} \quad \Big| \,\frac{\cdot S}{\cdot S}$$

$$= \frac{S}{1 - C_3 e^{-kSt}}$$

$$B(0) = \frac{S}{1 - C_3} = B_0 \Rightarrow 1 - C_3 = \frac{S}{B_0}$$

$$\Rightarrow \quad C_3 = 1 - \frac{S}{B_0}$$

$$S > B_0 \Rightarrow \frac{S}{B_0} > 1 \Rightarrow 1 - \frac{S}{B_0} < 0$$

$\Rightarrow$ Lsg. d. AWPs:

$$\boxed{B(t) = \frac{S}{1-\left(1-\frac{S}{B_0}\right) e^{-kSt}}}$$

mit $S > 0$
$S > B_0$
und $k > 0$