$\boxed{A1}$

a) $$\int \ln(2-3x)\,dx \underset{\boxed{\begin{matrix} z:=2-3x \\ \frac{dz}{dx}=-3 \\ dx=\frac{dz}{-3}\end{matrix}}}{=\!=} \int \ln(z)\,dz =$$

$$= -\frac{1}{3}\left[z\left(\ln z - 1\right)\right] \underset{R.S.}{=} -\frac{1}{3}(2-3x)\left(\ln(2-3x)-1\right) + C =$$

$$= \left(\frac{2}{3}-x\right)\left[1-\ln(2-3x)\right] + C$$

b) $$\int \tan(3x-4)\,dx \underset{\boxed{\begin{matrix} z:=3x-4 \\ \frac{dz}{dx}=3 \\ dx=\frac{dz}{3}\end{matrix}}}{=\!=} -\frac{1}{3}\int \frac{-\sin z}{\cos z}\,dz = -\frac{1}{3}\ln|\cos z| + C$$

$$\underset{R.S.}{=} -\frac{1}{3}\ln|\cos(3x-4)| + C$$

c) $$\int \sin x \sqrt{1-\cos x}\,dx \underset{\boxed{\begin{matrix} z:=1-\cos x \\ \frac{dz}{dx}=\sin x \\ dx=\frac{dz}{\sin x}\end{matrix}}}{=\!=} \int \sqrt{z}\,dz = \int z^{1/2}\,dz =$$

$$= \frac{2}{3}z^{\frac{3}{2}} + C \underset{R.S.}{=} \frac{2}{3}(1-\cos x)^{3/2} + C$$

e) $\int x \sqrt{2x+3}\, dx \;=\; \frac{1}{2}\int \left(\frac{z-3}{2}\right)\sqrt{z}\, dz =$

$$\boxed{x = \frac{z-3}{2}} \;\Longleftrightarrow\; \boxed{\begin{aligned} z &:= 2x+3 \\ \frac{dz}{dx} &= 2 \\ dx &= \frac{dz}{2} \end{aligned}}$$

$$= \frac{1}{4}\int z^{3/2}\, dz - \frac{3}{4}\int z^{1/2}\, dz = \frac{1}{4}\frac{2}{5} z^{5/2} - \frac{3}{4}\frac{2}{3} z^{3/2} + C$$

$$= \frac{1}{10} z^{5/2} - \frac{1}{2} z^{3/2} + C = \frac{1}{5} z^{3/2}\left(\frac{z}{2} - \frac{5}{2}\right) + C$$

$$\underset{R.S.}{=} \frac{1}{5}(2x+3)^{3/2}\left(\frac{1}{2}(2x+3) - \frac{5}{2}\right) + C = \frac{1}{5}(2x+3)^{3/2}(x-1) + C$$

j) $\int x^4 \sqrt{3x^5 - 4}\, dx \;=\; \frac{1}{15}\int \sqrt{z}\, dz =$

$$\boxed{\begin{aligned} z &:= 3x^5 - 4 \\ \frac{dz}{dx} &= 15x^4 \\ dx &= \frac{dz}{15x^4} \end{aligned}}$$

$$= \frac{1}{15}\frac{2}{3} z^{3/2} = \frac{2}{45}(3x^5 - 4)^{3/2} + C$$

u) $\int \cos^3(2x-1)\,dx \;=\; \frac{1}{2}\int \cos^3 z\,dz =$

$$\boxed{\begin{array}{l} z := 2x-1 \\ \frac{dz}{dx} = 2 \\ dx = \frac{dz}{2} \end{array}}$$

$$= \frac{1}{2}\int \underbrace{\cos^2 z}_{v}\,\underbrace{\cos z}_{u'}\,dz = \frac{1}{2}\left(\cos^2 z \sin z - \int \sin z \cdot 2\cos z(-\sin z)\,dz\right)$$

$$= \frac{1}{2}\sin z\cos^2 z + \frac{1}{2}\cdot 2\int \sin^2 z \cdot \cos z\,dz \;=$$

$$\boxed{\begin{array}{l} u := \sin z \\ \frac{du}{dz} = \cos z \\ dz = \frac{du}{\cos z} \end{array}}$$

$$= \frac{1}{2}\sin z\cos^2 z + \int u^2\,du = \frac{1}{2}\sin z\cos^2 z + \frac{1}{3}u^3 + C$$

$$\underset{R.S}{=} \frac{1}{2}\sin z\cos^2 z + \frac{1}{3}\sin^3 z + C = \frac{1}{2}\sin z\left(1-\sin^2 z\right) + \frac{1}{3}\sin^3 z + C$$

$$= \frac{1}{2}\sin z - \frac{1}{2}\sin^2 z + \frac{1}{3}\sin^3 z + C = \frac{1}{2}\sin z - \frac{1}{6}\sin^3 z + C$$

$$= \frac{1}{2}\sin z\left(1-\frac{1}{3}\sin^2 z\right) + C \underset{R.S.}{=} \frac{1}{2}\sin(2x-1)\left[1-\frac{1}{3}\sin^2(2x-1)\right] + C$$

k) $\int (2-7x)x^{1/3}dx = 2\int x^{1/3}dx - 7\int x^{7/3}dx =$

$= \frac{3}{2}x^{4/3} - 3x^{7/3} + C$

l) $\int e^{-2x}\cos\left(\frac{x}{2}\right)dx \quad = \quad 2\int \underbrace{e^{-4z}}_{u'}\underbrace{\cos z}_{v}\, dz =$

$x = 2z \quad \Leftrightarrow \quad \boxed{\begin{aligned} z &:= \frac{x}{2} \\ \frac{dz}{dx} &= \frac{1}{2} \\ dx &= 2dz \end{aligned}}$

$= 2\left(-\frac{1}{4}e^{-4z}\cos z + \frac{1}{4}\int e^{-4z}(-\sin z)dz\right) =$

$= -\frac{1}{2}e^{-4z}\cos z - \frac{1}{2}\int \underbrace{e^{-4z}}_{u'}\underbrace{\sin z}_{V}\, dz =$

$= -\frac{1}{2}e^{-4z}\cos z - \frac{1}{2}\left(-\frac{1}{4}e^{-4z}\sin z + \frac{1}{4}\int e^{-4z}\cos z\, dz\right)$

$= -\frac{1}{2}e^{-4z}\cos z + \frac{1}{8}e^{-4z}\sin z - \frac{1}{8}\int e^{-4z}\cos z\, dz$

$\Rightarrow 2\int e^{-4z}\cos z\, dz = -\frac{1}{2}e^{-4z}\cos z + \frac{1}{8}e^{-4z}\sin z.$

$- \frac{1}{8}\int e^{-4z}\cos z\, dz$

$\Rightarrow \frac{17}{8}\int e^{-4z}\cos z\, dz = -\frac{1}{8}e^{-4z}\cos z + \frac{1}{8}e^{-4z}\sin z + C_1 \quad \Big| : \frac{17}{8}$

$\Rightarrow \int e^{-4z}\cos z\, dz = -\frac{4}{17}e^{-4z}\cos z + \frac{1}{17}e^{-4z}\sin z + C_2$

$\Rightarrow 2\int e^{-4z}\cos z\, dz = -\frac{18}{17}e^{-4z}\cos z + \frac{2}{17}e^{-4z}\sin z + C_2$

$\underset{R.S.}{=} \cancel{2}\int e^{-2x}\cos\left(\frac{x}{2}\right)\frac{dx}{\cancel{2}} = -\frac{8}{17}e^{-2x}\cos\left(\frac{x}{2}\right) + \frac{2}{17}e^{-2x}\sin\left(\frac{x}{2}\right) + C_2$

$= \frac{2}{17}e^{-2x}\left(\sin\left(\frac{x}{2}\right) - 4\cos\left(\frac{x}{2}\right)\right) + C_2$

p) $\int \frac{e^x}{e^{2x}+1}dx \;=\!=\; \int \frac{dz}{1+z^2} =$

$$\boxed{\begin{aligned} z &:= e^x \\ \frac{dz}{dx} &= e^x \\ dx &= \frac{dz}{e^x} \end{aligned}}$$

$= \arctan(z) + C \underset{R.S.}{=} \arctan(e^x) + C$

q) $\int \frac{x}{\sqrt{1-x}}dx \;=\!=\; -\int \frac{1-z}{\sqrt{z}}dz =$

$$\boxed{\begin{aligned} z &:= 1-x \\ \frac{dz}{dx} &= -1 \\ dx &= -dz \end{aligned}}$$

$= -\int z^{-1/2}dz + \int z^{1/2}dz = -2z^{1/2} + \frac{2}{3}z^{3/2} + C$

$= -2z^{1/2}\left(1-\frac{1}{3}z\right)+C \underset{R.S.}{=} -2(1-x)^{1/2}\left(1-\frac{1}{3}+\frac{x}{3}\right)+C$

$= -2(1-x)^{1/2}\left(\frac{2}{3}+\frac{x}{3}\right)+C = -\frac{2}{3}(1-x)^{1/2}(x+2)+C$

$$\text{v)}\; I := \int \frac{2-x}{\sqrt{1-x^2}}\,dx = \underbrace{2\int \frac{dx}{\sqrt{1-x^2}}}_{=:\, I_1} - \underbrace{\int \frac{x}{\sqrt{1-x^2}}\,dx}_{=:\, I_2}$$

$$I_1 = 2\arcsin x$$

$$I_2 = -\int \frac{x}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\int \frac{dz}{\sqrt{z}} = \frac{1}{2}\int z^{-1/2}\,dz =$$

$$\boxed{\begin{array}{l} z := 1-x^2 \\ \frac{dz}{dx} = -2x \\ dx = -\frac{1}{2x}\,dz \end{array}}$$

$$= \frac{1}{2}\cdot 2\, z^{1/2} \underset{\text{R.S.}}{=} (1-x^2)^{1/2}$$

$$\Longrightarrow I = 2\arcsin x + \sqrt{1-x^2} + C$$

$$\text{i)}\; \int x^2 (x^3+1)^3\,dx = \frac{1}{3}\int z^3\,dz =$$

$$\boxed{\begin{array}{l} z := x^3+1 \\ \frac{dz}{dx} = 3x^2 \\ dx = \frac{dz}{3x^2} \end{array}}$$

$$= \frac{1}{3}\cdot\frac{1}{4}\, z^4 + C \underset{\text{R.S.}}{=} \frac{1}{12}(x^3+1)^4 + C$$

$\boxed{\text{A5}}$

a) $$\int \frac{dx}{\sqrt{a^2-x^2}} = \int \frac{a\cos u}{\sqrt{a^2-a^2\sin^2 u}}\,du =$$

$$\boxed{\begin{aligned} &x = a\sin u \\ &\frac{dx}{du} = a\cos u \\ &dx = a\cos u\,du \end{aligned}} \Longleftrightarrow \quad u = \arcsin\left(\frac{x}{a}\right)$$

$$= \int \frac{a\cos u}{a\sqrt{1-\sin^2 u}}\,du = \int \frac{\cancel{a\cos u}}{\cancel{a\cos u}}\,du = \int 1\,du$$

$$= u + C \underset{\text{R.S.}}{=} \arcsin\left(\frac{x}{a}\right) + C$$

b) $$\int \sqrt{4-9x^2}\,dx = \int \sqrt{9\left(\frac{4}{9}-x^2\right)}\,dx = 3\int \sqrt{\frac{4}{9}-x^2}\,dx$$

$$= 3\int \sqrt{\frac{4}{9}-\frac{4}{9}\sin^2 u}\cdot\frac{2}{3}\cos u\,du = \cancel{3}\cdot\frac{2}{\cancel{3}}\int \sqrt{\frac{4}{9}(1-\sin^2 u)}\cos u\,du$$

$$\boxed{\begin{aligned} &x = \frac{2}{3}\sin u \\ &\frac{dx}{du} = \frac{2}{3}\cos u \\ &dx = \frac{2}{3}\cos u\,du \end{aligned}}$$

$$= \frac{4}{3}\int \cos^2 u\,du = \frac{4}{3}\left(\frac{1}{2}u + \frac{1}{4}\sin(2u)\right) + C$$

$$= \frac{2}{3}u + \frac{1}{3}\sin(2u) + C = \frac{2}{3}u + \frac{1}{3}\cdot 2\sin u\cos u + C$$

$$= \frac{2}{3}u + \frac{2}{3}\sin u\sqrt{1-\sin^2 u} + C$$

$$\underset{\text{R.S.}}{=} \frac{2}{3}\arcsin\left(\frac{3}{2}x\right) + \frac{2}{3}\sin\left(\arcsin\left(\frac{3}{2}x\right)\right)\sqrt{1-\sin^2\left(\arcsin\left(\frac{3}{2}x\right)\right)} + C$$

$$= \frac{2}{3}\arcsin\left(\frac{3}{2}x\right) + \cancel{\frac{2}{3}}\cdot\cancel{\frac{3}{2}}x\sqrt{1-\left(\frac{3}{2}x\right)^2} = \frac{2}{3}\arcsin\left(\frac{3}{2}x\right) + x\sqrt{1-\frac{9}{4}x^2} + C$$

$$= \frac{2}{3}\arcsin\left(\frac{3}{2}x\right) + x\sqrt{\frac{4-9x^2}{4}} + C = \frac{2}{3}\arcsin\left(\frac{3}{2}x\right) + \frac{x}{2}\sqrt{4-9x^2} + C$$

c) $$\int \frac{dx}{x^2\sqrt{a^2-x^2}} = \int \frac{a\cos u}{a^2\sin^2 u\sqrt{a^2-\sin^2 u\cdot a^2}}\,du =$$

$$\boxed{\begin{aligned} &x = a\sin u \\ &\frac{dx}{du} = a\cos u \\ &dx = a\cos u\,du \end{aligned}}$$

$$= \int \frac{a\cos u}{a^2\sin^2 u\; a\sqrt{1-\sin^2 u}}\,du = \int \frac{a\cos u}{a^2\sin^2 u\cdot a\cos u}\,du =$$

$$= \frac{1}{a^2}\int \frac{du}{\sin^2 u} = \frac{1}{a^2}(-\cot u) + C = -\frac{1}{a^2}\,\frac{\cos u}{\sin u} + C =$$

$$= -\frac{1}{a^2}\,\frac{\sqrt{1-\sin^2 u}}{\sin u} + C \underset{R.S.}{=} -\frac{1}{a^2}\cdot\frac{\sqrt{1-\sin^2\left(\arcsin\left(\frac{x}{a}\right)\right)}}{\sin\left(\arcsin\left(\frac{x}{a}\right)\right)} + C =$$

$$= -\frac{1}{a^2}\,\frac{\sqrt{1-\frac{x^2}{a^2}}}{\frac{x}{a}} + C = -\frac{1}{a^2}\cdot\frac{\sqrt{\frac{a^2-x^2}{a^2}}}{\frac{x}{a}} + C =$$

$$= -\frac{1}{\cancel{a^2}_{\,a}}\,\frac{\frac{\sqrt{a^2-x^2}}{a}}{\frac{x}{\cancel{a}}} = -\frac{1}{a^2}\cdot\frac{\sqrt{a^2-x^2}}{x} + C$$