$\boxed{A88}$ a) $a_n = \frac{2n+1}{n} = 2 + \frac{1}{n}$

Es gilt: $2 < a_n \leq 3 \quad \forall n \in \mathbb{N} \Rightarrow (a_n)$ beschränkt.

Monotonie: $a_{n+1} - a_n = 2 + \frac{1}{n+1} - 2 - \frac{1}{n} = \frac{n-(n+1)}{(n+1)n} = -\frac{1}{n(n+1)} < 0 \quad \forall n$

$\Rightarrow (a_n)$ streng mon. $\downarrow$.

$\lim\limits_{n\to\infty} \left(2 + \frac{1}{n}\right) = 2.$

$\boxed{A90}$ c) $a_n = \frac{\left(\sqrt{n^2+1}-n\right)\left(\sqrt{n^2+1}+n\right)}{\sqrt{n^2+1}+n} = \frac{n^2+1-n^2}{\sqrt{n^2+1}+n} = \frac{1}{\sqrt{n^2+1}+n} \xrightarrow{(n\to\infty)} 0$

$b_n = \frac{\left(\sqrt{n+1}-\sqrt{n-2}\right)\left(\sqrt{n+1}+\sqrt{n-2}\right)}{\sqrt{n+1}+\sqrt{n-2}} = \frac{n+1-(n-2)}{\sqrt{n+1}+\sqrt{n-2}} = \frac{3}{\sqrt{n+1}+\sqrt{n-2}} \xrightarrow{(n\to\infty)} 0$

d) $\lim\limits_{n\to\infty} \left(1+\frac{2}{n}\right)^{100} = \left[\lim\limits_{n\to\infty}\left(1+\frac{2}{n}\right)\right]^{100} = 1$

$\lim\limits_{n\to\infty} \left(1+\frac{3}{n}\right)^{2n} = \left[\lim\limits_{n\to\infty}\left(1+\frac{3}{n}\right)^{n}\right]^{2} = \left(e^{3}\right)^{2} = e^{6}$

$\lim\limits_{n\to\infty} \left(1-\frac{1}{3n}\right)^{2n} = \left[\lim\limits_{n\to\infty}\left(1-\frac{1}{3n}\right)^{n}\right]^{2} = \left(e^{-1/3}\right)^{2} = e^{-2/3}$

$\lim\limits_{n\to\infty} \left(1+\frac{4}{n-1}\right)^{n+4} = \lim\limits_{n\to\infty}\left(1+\frac{4}{n-1}\right)^{n} \cdot \lim\limits_{n\to\infty}\left(1+\frac{4}{n-1}\right)^{4}$

$= e^{4} \cdot 1 = e^{4}$

e) $a_n = \frac{3^n+6^n}{3^n+6^{n+1}} = \frac{3^n+2^n\cdot 3^n}{3^n+2^n\cdot 3^n\cdot 6} = \frac{\cancel{3^n}(1+2^n)}{\cancel{3^n}(1+2^n\cdot 6)} = \frac{\cancel{2^n}\left(\frac{1}{2^n}+1\right)}{\cancel{2^n}\left(\frac{1}{2^n}+6\right)} \xrightarrow{(n\to\infty)} \frac{1}{6}$

$\boxed{A\,91}$ $a_n = a_1 + (n-1)d$

$$a_5 = a_1 + 4 \cdot d = a_1 + 4 \cdot (-2) \overset{!}{=} 22$$

$$\Rightarrow a_1 = 30 \Rightarrow a_n = 30 + (n-1) \cdot d$$

$$\Rightarrow a_8 = 30 + 7(-2) = 16 \; ; \; a_{100} = 30 + 99(-2) = -168$$

$\boxed{A\,92}$ $b_n = b_1 \cdot q^{n-1}$

$$b_3 = b_1 \cdot q^2 = 2 \quad (1) \qquad b_5 = b_1 \cdot q^4 = 8 \quad (2)$$

$$\text{"}(2):(1)\text{"} \Rightarrow q^2 = 4 \Rightarrow |q| = 2 \underset{(1)}{\Rightarrow} b_1 = \frac{1}{2}$$

$$\Rightarrow b_n = \frac{1}{2} \cdot 2^{n-1} = 2^{n-2}$$

$$\vee \left[b_n = \frac{1}{2}(-2)^{n-1} = (-1)^{n-1} \cdot \frac{2^n}{2} = (-1)^{n-1} 2^{n-1} = (-2)^{n-1} \right]$$

$\boxed{A\,94}$ $2 - \frac{2}{a} + \frac{2}{a^2} - + \ldots$

geom. Reihe mit $q = \dfrac{-\frac{2}{a}}{2} = -\dfrac{1}{a}$

Reihe konv. $\Leftrightarrow$ $|q| = \left|-\frac{1}{a}\right| = \frac{1}{|a|} < 1 \Leftrightarrow |a| > 1$

$$\Leftrightarrow a > 1 \;\vee\; a < -1$$

$$S_\infty = \frac{2}{1 + \frac{1}{a}} = \frac{2}{\frac{a+1}{a}} = \frac{2a}{a+1}$$

$\boxed{A\ 96}$

$a_1 = 7$

$a_{n+1} = a_n + 2$

$a_2 = a_1 + 2 = 9$

$a_3 = a_2 + 2 = a_1 + 2 + 2 = 11$

$a_4 = a_3 + 2 = a_1 + 2 + 2 + 2 = 13$

Vermutung: $\boxed{a_n = 7 + (2n-1)}$

(müsste durch vollst. Ind. bewiesen werden)

$\boxed{A\ 97}$

b) $\dfrac{a_{n+1}}{a_n} = \dfrac{2^n \cdot 3}{2^{n-1} \cdot 3} = 2 \Longrightarrow \boxed{a_{n+1} = 2a_n \quad ; \quad a_1 = 3}$

a) $a_{n+1} - a_n = (n+1)^2 + (n+1) - 2 - (n^2 + n - 2)$

$= \cancel{n^2} + 2n + 1 + \cancel{n} + 1 \cancel{- 2} \cancel{- n^2} \cancel{- n} \cancel{+ 2}$

$\Longrightarrow \boxed{a_{n+1} = a_n + 2n + 2 \quad ; \quad a_1 = 0}$