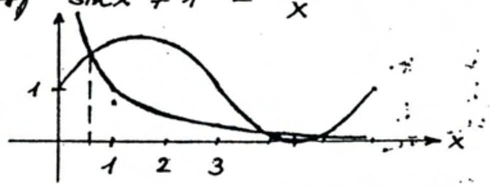
Lösungen Blatt # 10

84) $\sin x + 1 = \frac{1}{x}$



es gibt unendlich viele positive Lösungen; es gibt keine negativen Lösungen

Rohwert zeichnerisch: $x_0 = 0.6$

$$x_1 = 0.6 - \frac{\sin 0.6 + 1 - \frac{1}{0.6}}{\cos 0.6 + \left(\frac{1}{0.6}\right)^2}$$

$$= 0.6 - \frac{-0.10202}{3.60311} =$$

$$= 0.6283$$

$x_2 = 0.6294$; $x_3 = 0.6294$

für große x gilt näherungsweise $\frac{1}{x} \approx 0$; die Gleichung wird zu

$\sin x = -1$

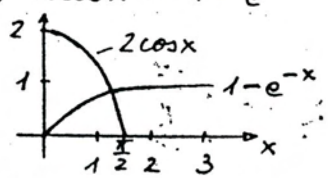
$x = \frac{3}{2}\pi + k \cdot 2\pi \qquad k = 1, 2, 3 \ldots$

Rohwert rechnerisch: $\sin x \approx x$

$x + 1 = \frac{1}{x}$; $x^2 + x - 1 = 0$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+4}}{2} = \begin{cases} 0.6180 \\ -1.618 \text{ entfällt.} \end{cases}$$

85) $2\cos x = 1 - e^{-x}$



Rohwert: $x_0 = 1.2$

rechnerisch:

$$2\left(1 - \frac{x^2}{2!}\right) = 1 - \left(1 - x + \frac{x^2}{2!}\right)$$

$-\frac{1}{2}x^2 - x + 2 = 0$

$x_{1,2} = -1 \, (\underset{-}{+}) \sqrt{5} = 1.236$

Newton Iteration mit $x_0 = 1.2$

$x_1 = 1.21196$

$x_2 = 1.21195$

es gibt unendlich viele positive Lösungen; für große x gilt $e^{-x} \approx 0$

$2\cos x = 1$; $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3} + k \cdot 2\pi \qquad k = 1, 2, 3$

86) a) $\lim\limits_{x \to 0} \frac{x \cdot \sin 2x}{e^x - 2 + e^{-x}} \qquad "\frac{0}{0}"$

D.H. $= \lim\limits_{x \to 0} \frac{\sin 2x + x \cdot 2\cos 2x}{e^x - e^{-x}} \qquad "\frac{0}{0}"$

D.H. $= \lim\limits_{x \to 0} \frac{2\cos 2x + 2\cos 2x - 4x \sin 2x}{e^x + e^{-x}}$

$= 2$

b) $\lim\limits_{x \to 3} \frac{3x(1 - e^{x-3})}{x - 3e^{x-3}} \qquad "\frac{0}{0}"$

D.H. $= \lim\limits_{x \to 3} \frac{3(1 - e^{x-3}) + 3x(-e^{x-3})}{1 - 3e^{x-3}} = \frac{9}{2}$

c) $\lim\limits_{x \to 0} \frac{\ln \sin x}{\ln \tan x} \qquad "\frac{-\infty}{-\infty}"$

D.H. $= \lim\limits_{x \to 0} \frac{\frac{1}{\sin x} \cdot \cos x}{\frac{1}{\tan x} \cdot \frac{1}{\cos^2 x}}$

$= \lim\limits_{x \to 0} \frac{\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} \cdot \cos^3 x}{1}$

$= \lim\limits_{x \to 0} \cos^2 x = 1$

d) $\lim\limits_{x \to \infty} (5+x) \ln\left(1 + \frac{5}{x}\right) \qquad "\infty \cdot 0"$

$= \lim\limits_{x \to \infty} \frac{\ln\left(1 + \frac{5}{x}\right)}{\frac{1}{5+x}} \qquad "\frac{0}{0}"$

D.H. $= \lim\limits_{x \to \infty} \frac{\frac{1}{1+5/x} \cdot \left(-\frac{5}{x^2}\right)}{-(5+x)^{-2}}$

$= \lim\limits_{x \to \infty} \frac{(5+x)^2 \cdot 5}{\left(1 + \frac{5}{x}\right) \cdot x^2}$

$= \lim\limits_{x \to \infty} \frac{5(x^2 + 10x + 25)}{x^2 + 5x}$

D.H. $= \lim\limits_{x \to \infty} \frac{5(2x + 10)}{2x + 5}$ D.H. $= \lim\limits_{x \to \infty} \frac{10}{2} = 5$

e) $\lim\limits_{x \to 2} \left(\frac{1}{\ln(x-1)} - \frac{1}{x-2}\right) \qquad "\infty - \infty"$

$= \lim\limits_{x \to 2} \frac{x - 2 - \ln(x-1)}{(x-2)\ln(x-1)} \qquad "\frac{0}{0}"$

D.H. $= \frac{1 - \frac{1}{x-1}}{\ln(x-1) + (x-2)\frac{1}{x-1}} \qquad \frac{0}{0}$

D.H. $= \frac{\frac{1}{(x-1)^2}}{\frac{1}{x-1} + \frac{1}{(x-1)^2}} = \frac{1}{2}$

f) $\lim\limits_{x \to 1} \frac{\ln(x^2 + 2x - 2)}{x - 1} = 4$

g) $\lim\limits_{x \to 0} \left(\frac{e^{-x}}{\sin x} - \frac{1}{x}\right) = -1$

h) $\lim\limits_{x \to \infty} \frac{\sqrt{1 + 2x^2}}{x + 1} = \sqrt{2}$

i) $\lim\limits_{x \to 1} \frac{x - 1 - \ln x}{(x-1)^2} = \frac{1}{2}$

k) $\lim\limits_{x \to 0} (e^{-3x} - 1)\cot x = -3$

l) $\lim\limits_{x \to 2} \left(\frac{1}{x-2} - \frac{1}{e^x - e^2}\right) \longrightarrow \infty$

$\lim \ln \frac{\sin x}{} = \lim \ln \ldots$

87) $\lim\limits_{f \to \infty} u(f) = "\frac{\infty}{\infty}"$

$\lim\limits_{f \to \infty} u(f) = \lim\limits_{f \to \infty} \frac{8\pi h \cdot 3f^2}{c^3 \frac{h}{kT} e^{\frac{hf}{kT}}}$

$= \lim\limits_{f \to \infty} \frac{8\pi h \cdot 3 \cdot 2f}{c^3 \left(\frac{h}{kT}\right)^2 e^{\frac{hf}{kT}}} =$

$= \lim\limits_{f \to \infty} \frac{8\pi h \cdot 3 \cdot 2}{c^3 \left(\frac{h}{kT}\right)^3 e^{\frac{hf}{kT}}} = 0$

$\lim\limits_{f \to 0} u(f) = "\frac{0}{0}"$

$= \lim\limits_{f \to 0} \frac{8\pi h \, 3f^2}{c^3 \frac{h}{kT} e^{\frac{hf}{kT}}} = 0$