Lösungen Blatt 9

79b) $g(x) = \sin x \qquad g'(x) = \cos x$

$g''(x) = -\sin x \qquad g'''(x) = -\cos x$

$$\sin x = \sin\frac{\pi}{2} + \cos\frac{\pi}{2}\left(x-\frac{\pi}{2}\right) - \frac{1}{2!}\sin\frac{\pi}{2}\left(x-\frac{\pi}{2}\right)^2 - \frac{1}{3!}\cos\frac{\pi}{2}\left(x-\frac{\pi}{2}\right)^3 + + - - \dots$$

$$\sin x = 1 - \frac{1}{2!}\left(x-\frac{\pi}{2}\right)^2 + \frac{1}{4!}\left(x-\frac{\pi}{2}\right)^4 - \frac{1}{6!}\left(x-\frac{\pi}{2}\right)^6$$

80) $$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1\cdot 1}{2\cdot 4}x^2 + \frac{1\cdot1\cdot3}{2\cdot4\cdot6}x^3 - \frac{1\cdot1\cdot3\cdot5}{2\cdot4\cdot6\cdot8}x^4 + \dots$$

$$\int_0^{0.5}\sqrt{1+x}\,dx = \int_0^{0.5}\left(1+\frac{1}{2}x - + \dots\right)dx$$

$$\approx x + \frac{1}{2}\cdot\frac{x^2}{2} - \frac{1\cdot1}{2\cdot4}\cdot\frac{x^3}{3} + \frac{1\cdot1\cdot3}{2\cdot4\cdot6}\cdot\frac{x^4}{4} - \frac{1\cdot1\cdot3\cdot5}{2\cdot4\cdot6\cdot8}\cdot\frac{x^5}{5}\Big|_0^{0.5}$$

$$= \frac{1}{2} + \frac{1}{16} - \frac{1}{192} + \frac{1}{2^{10}} - \frac{1}{2^{12}}$$

$$\int_0^{0.5}\sqrt{\%}\,dx < \frac{1}{2}+\frac{1}{16}-\frac{1}{192}+\frac{1}{2^{10}} = 0.558268$$

$$\int_0^{0.5}\sqrt{\%}\,dx > \frac{1}{2}+\frac{1}{16}-\frac{1}{192}+\frac{1}{2^{10}}-\frac{1}{2^{12}} = 558024$$

81a) $$\int_0^1\frac{1}{5+x^2}\,dx = \frac{1}{\sqrt5}\arctan\frac{x}{\sqrt5}\Big|_0^1 = \frac{1}{\sqrt5}\arctan\frac{1}{\sqrt5} = 0.188068$$

$$\int_0^1\frac{1}{5+x^2}\,dx \approx \int_0^1\frac{1}{5}\left(1-\frac{x^2}{5}+\frac{x^4}{25}-\frac{x^6}{125}\right)dx$$

$$= \frac{1}{5}\left(x - \frac{x^3}{15} + \frac{x^5}{125} - \frac{x^7}{875}\right)\Big|_0^1$$

$$= \frac{1}{5}\left(1 - \frac{1}{15} + \frac{1}{125} - \frac{1}{875}\right) = 0.188038$$

b) $$\int_0^{0.5}\frac{x^3+x}{x^2+2}\,dx = \int_0^{0.5}\left(x - \frac{x}{x^2+2}\right)dx$$

$$= \frac{x^2}{2}\Big|_0^{0.5} - \int_0^{0.5}\frac{x\cdot 2}{(x^2+2)\cdot 2}\,dx$$

$$= \frac{x^2}{2}\Big|_0^{0.5} - \frac{1}{2}\ln(x^2+2)\Big|_0^{0.5}$$

$$= \frac{1}{8} - \frac{1}{2}\ln\frac{9}{4} + \frac{1}{2}\ln 2$$

$$= \frac{1}{8} + \frac{1}{2}\ln\frac{8}{9} = 0.066108$$

$$\int_0^{0.5}\frac{x^3+x}{x^2+2}\,dx \approx \int_0^{0.5}\left(\frac{x}{2}+\frac{x^3}{4}-\frac{x^5}{8}+\frac{x^7}{12}\right)dx$$

$$= \frac{x^2}{4} + \frac{x^4}{16} - \frac{x^6}{48} + \frac{x^8}{96}\Big|_0^{0.5}$$

$$= \frac{1}{16} + \frac{1}{256} - \frac{1}{3072} + \frac{1}{24576}$$

$$= 0.066121$$

$$\int_0^{0.5}\%\,dx > 0.06608$$

$$\int_0^{0.5}\%\,dx < 0.066121$$

c) $$\int_0^1\frac{x^3-x^2+x-1}{x+2}\,dx$$

$$= \int_0^1\left(x^2-3x+7-\frac{15}{x+2}\right)dx$$

$$= \frac{x^3}{3} - 3\frac{x^2}{2} + 7x - 15\ln(x+2)\Big|_0^1$$

$$= \frac{1}{3} - \frac{3}{2} + 7 - 15\ln 3 + 15\ln 2$$

$$= -0.248643$$

$$\int_0^1\%\,dx \approx \int_0^1\left(-\frac{1}{2}+\frac{3}{4}x-\frac{7}{8}x^2+\frac{15}{16}x^3 - \frac{15}{32}x^4+\frac{15}{64}x^5-\frac{15}{128}x^6\right)dx$$

$$\int_0^1\%\,dx \approx -\frac{1}{2}x + \frac{3}{8}x^2 - \frac{7}{24}x^3 + \frac{15}{64}x^4 - \frac{15}{160}x^5 + \frac{15}{384}x^6 - \frac{15}{896}x^7\Big|_0^1$$

$$= -\frac{1}{2} + \frac{3}{8} - \frac{7}{24} + \frac{15}{64} - \frac{15}{160} + \frac{15}{384} - \frac{15}{896} = -0.25392$$

$$\int_0^1\%\,dx > -0.25392$$

$$\int_0^1\%\,dx < -0.23698$$

82a) $$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$$

$$\int_0^2\frac{\sin x}{x}\,dx = \int_0^2\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\frac{x^8}{9!}\right)dx$$

$$= x - \frac{x^3}{3\cdot3!} + \frac{x^5}{5\cdot5!} - \frac{x^7}{7\cdot7!} + \frac{x^9}{9\cdot9!}\Big|_0^2$$

$$= 2 - \frac{8}{18} + \frac{32}{600} - \frac{128}{35280} + \frac{512}{3265920}$$

$$= 1.605418$$

$$1.605261 < \int_0^2\frac{\sin x}{x}\,dx < 1.605418$$

b) $$\frac{\cos x}{\sqrt{1-x^2}} = \cos x\,(1-x^2)^{-1/2}$$

$$= \left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots\right)\cdot\left(1+\frac{1}{2}x^2+\frac{1\cdot3}{2\cdot4}x^4+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}x^6+\dots\right)$$

$$= 1 + \frac{1}{2}x^2 + \frac{1\cdot3}{2\cdot4}x^4 + \frac{1\cdot3\cdot5}{2\cdot4\cdot6}x^6 - \frac{x^2}{2} - \frac{x^4}{4} - \frac{1\cdot3}{2\cdot2\cdot4}x^6 + \frac{x^4}{4!} + \frac{1}{4!\,2}x^6 - \frac{1}{6!}x^6$$

$$= 1 + \frac{1}{6}x^4 + \frac{13}{90}x^6$$

$$\int_0^{1/2}\%\,dx \approx \int_0^{1/2}\left(1+\frac{1}{6}x^4+\frac{13}{90}x^6\right)dx$$

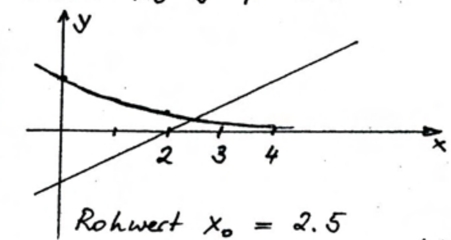
$$= x + \frac{1}{30}x^5 + \frac{13}{630}x^7\Big|_0^{1/2}$$

$$= \frac{1}{2} + \frac{1}{960} + \frac{13}{630\cdot128} = 0.50120$$

83) $$e^{0.5x}\left(\frac{x}{2}-1\right) = 1$$

$$\frac{x}{2} - 1 = e^{-0.5x}$$

Rohwert x_0 graphisch



Rohwert $x_0 = 2.5$

Gleichung besiht nur eine Lösung

$$x_1 = 2.5 - \frac{e^{1.25}(1.25-1)-1}{\frac{1}{2}\cdot e^{1.25}\cdot 1.25}$$

$$= 2.5 + 0.0584 = 2.5584$$

$$x_2 = 2.5584 - 0.00147 = 2.55693$$

$$x_3 = 2.55693 - 0.0000009$$

$$x_3 = 2.5569291$$

Rohwert rechnerisch:

Methode 1: $$\underbrace{e^{0.5x}}_{\text{Taylorpolynom 1. Grades}}\left(\frac{x}{2}-1\right) - 1 = 0$$

$$\left(1+\frac{1}{2}x\right)\left(\frac{1}{2}x-1\right) - 1 = 0$$

$$\frac{x^2}{4} - 1 = 1;\quad x^2 = 8;\quad x_0 = \sqrt8 \approx 2.8$$

Methode 2: $$\frac{x}{2} - 1 = e^{-\frac{1}{2}x} = e^{-1} - \frac{1}{2}e^{-1}(x-2)$$

Taylorpol. an der Stelle $x_0 = 2$

$$x_0 = \frac{2e^{-1}+1}{e^{-1}+1}\cdot 2 = 2.539$$