Lösungen Blatt 9

75a) $e^{x^2} = e^4$

$e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots$

$-\infty < x < \infty$

b) $\cosh x - \cos x = \frac{1}{2}(e^x + e^{-x}) - \cos x$

$= \frac{1}{2}\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)$

$+ \frac{1}{2}\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - + \cdots\right)$

$- 1 + \frac{x^2}{2!} - \frac{x^4}{4!} + - \cdots$

$= x^2 + \frac{2}{6!}x^6 + \frac{2}{10!}x^{10} + \cdots$

Bem: $\cosh x - \cos x$ ist achsen- [illegible] ... Achse.

also enthält die Taylorreihe nur x-Potenzen mit gerader Hochzahl

$-\infty < x < \infty$

c) $\frac{x^2}{\sqrt{1+x^2}} = x^2 \cdot (1+x^2)^{-1/2}$

$= x^2\left(1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - + \cdots\right)$

$= x^2 - \frac{1}{2}x^3 + \frac{1 \cdot 3}{2 \cdot 4}x^4 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^5 + \cdots$

$|x| < 1$

d) $\left(\frac{1}{1-x^2} - 1\right) \cdot \frac{1}{x} - x^2$

$= (1 + x^2 + x^4 + x^6 + \cdots - 1)\frac{1}{x} - x^2$

$= x + x^3 + x^5 + x^7 + \cdots - x^2$

$= x - x^2 + x^3 + x^5 + x^7 + \cdots$

$|x| < 1$

76 a) $\frac{1 - \cos x}{x^2} = \frac{1}{x^2}(1 - \cos x)$

$= \frac{1}{x^2}\left(1 - 1 + \frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \cdots\right)$

$= \frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \frac{x^6}{8!} + \cdots$

$-\infty < x < \infty$

b) $\frac{x - \sin 3x}{x} = 1 - \frac{1}{x} \cdot \sin 3x$

$= 1 - \frac{1}{x}\left(3x - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - \cdots\right)$

$= -2 + \frac{9}{2}x^2 - \frac{81}{40}x^4 + \frac{243}{560}x^6 - \cdots$

$-\infty < x < \infty$

c) $\frac{1-x}{1+x} = (1-x) \cdot \frac{1}{1+x}$

$= (1-x)(1 - x + x^2 - x^3 + - \cdots)$

$= 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$

$= 1 - 2x + 2x^2 - 2x^3 + 2x^4 - 2x^5 + - \cdots$

$|x| < 1$

77a) $\frac{3}{2+x} = \frac{3}{2(1+\frac{x}{2})} = \frac{3}{2}\frac{1}{1+\frac{x}{2}}$

$= \frac{3}{2}\left(1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \frac{x^4}{16} - \cdots\right)$

$= \frac{3}{2} - \frac{3}{4}x + \frac{3}{8}x^2 - \frac{3}{16}x^3 + \frac{3}{32}x^4$

$\left|\frac{x}{2}\right| < 1$; $|x| < 2$

b) $\frac{1}{5+x^2} = \frac{1}{5(1+\frac{x^2}{5})}$

$= \frac{1}{5}\left(1 - \frac{x^2}{5} + \frac{x^4}{25} - \frac{x^6}{125} + - \cdots\right)$

$\left|\frac{x^2}{5}\right| < 1$; $|x| < \sqrt{5}$

c) $\frac{x^3+x}{x^2+2} = x - \frac{x}{x^2+2}$

$= x - x \cdot \frac{1}{2(1+\frac{x^2}{2})}$

$= x - x\frac{1}{2}\left(1 - \frac{x^2}{2} + \frac{x^4}{4} - \frac{x^6}{8} + \cdots\right)$

$= \frac{x}{2} + \frac{x^3}{4} - \frac{x^5}{8} + \frac{x^7}{16} - + \cdots$

$\left|\frac{x^2}{2}\right| < 1$; $|x| < \sqrt{2}$

d) $\frac{x^3 - x^2 + x - 1}{x+2} = x^2 - 3x + 7 - \frac{15}{x+2}$

$= x^2 - 3x + 7 - \frac{15}{2(1+\frac{x}{2})}$

$= x^2 - 3x + 7 - \frac{15}{2}\left(1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} \cdots\right)$

$= -\frac{1}{2} + \frac{3}{4}x - \frac{7}{8}x^2 + \frac{15}{16}x^3 - \frac{15}{32}x^4$

$+ \frac{15}{64}x^5 - \frac{15}{128}x^6 + \cdots$

$\left|\frac{x}{2}\right| < 1$; $|x| < 2$

78) $\frac{e^{-x}}{\sqrt{1+x}} = e^{-x} \cdot (1+x)^{-1/2}$

$= \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \cdots\right) \cdot$

$\left(1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \cdots\right)$

systematisches ausmultiplizieren ergibt:

$1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + - \cdots$

$- x + \frac{1}{2}x^2 - \frac{1 \cdot 3}{2 \cdot 4}x^3 + \cdots$

$+ \frac{1}{2}x^2 - \frac{1}{4}x^3 + \cdots$

$- \frac{1}{3!}x^3 + \cdots$

$= 1 - \frac{3}{2}x + \frac{11}{8}x^2 - \frac{53}{48}x^3 + \frac{115}{128}x^4$

$|x| < 1$

79) $f(x) = f''(x) = f^{IV}(x) = \cdots = e^{-x}$

$f'(x) = f'''(x) = f^{V}(x) = \cdots = -e^{-x}$

$f(1) = f''(1) = f^{IV}(1) = \cdots = e^{-1}$

$f'(1) = f'''(1) = f^{V}(1) = \cdots = -e^{-1}$

$f(x) = e^{-1} - e^{-1}(x-1) + \frac{e^{-1}}{2!}(x-1)^2$

$- \frac{e^{-1}}{3!}(x-1)^3 + \frac{e^{-1}}{4!}(x-1)^4 - + \cdots$

$e^{-1.2} = e^{-1} - 0.2 \cdot e^{-1} + \frac{e^{-1}}{2!}(0.2)^2$

$- \frac{e^{-1}}{3!}(0.2)^3 + - \cdots$

$e^{-1.2} = e^{-1}\left(1 - 0.2 + 0.02 - \frac{1}{750} + - \cdots\right)$

$e^{-1.2} > 0.30117$

$e^{-1.2} < 0.30166$