Lösungen Übungsblatt 8

71e) $\int \frac{3\sin x}{\sqrt{2+\cos x}}\,dx = \int -\frac{3}{\sqrt{u}}\,du$

$2+\cos x = u \qquad = -6\sqrt{2+\cos x}$

f) $\int \frac{x^2+1}{\sqrt{x+1}}\,dx = \int \frac{u^2-2u+2}{\sqrt{u}}\,du$

$x+1=u \qquad = \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + 4u^{1/2}$

$= \sqrt{x+1}\left(\frac{2}{5}(x+1)^2 - \frac{4}{3}(x+1) + 4\right)$

g) $\int \frac{1}{\sin^2 x \cot x}\,dx = \int \frac{\frac{1}{\sin^2 x}}{\cot x}\,dx$

$= -\ln|\cot x| = \ln|\tan x|$

b) $\int \frac{2x+10}{(x-1)(x+2)(x+3)}\,dx$

$\% = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3}$

$A = \lim\limits_{x\to 1} \frac{2x+10}{(x+2)(x+3)} = 1$

$B = \lim\limits_{x\to -2} \frac{2x+10}{(x-1)(x+3)} = -2$

$C = \lim\limits_{x\to -3} \frac{2x+10}{(x-1)(x+2)} = 1$

$\int \%\,dx = \int \frac{1}{x-1}\,dx + \int \frac{-2}{x+2}\,dx$

$+ \int \frac{1}{x+3}\,dx$

$= \ln|x-1| - 2\ln|x+2| + \ln|x+3|$

$\int \frac{5x+4}{(x-1)(x+2)^2}\,dx$

$\% = \frac{A}{x-1} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$

$A = \lim\limits_{x\to 1} \frac{5x+4}{(x+2)^2} = 1$

$C = \lim\limits_{x\to -2} \frac{5x+4}{x-1} = 2$

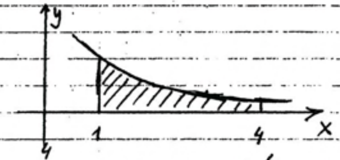
$x=0: \ \frac{B}{2} - 1 + \frac{1}{2} = -1 \ ; \ B = -1$

$\int \%\,dx = \int \frac{1}{x-1}\,dx + \int \frac{-1}{x+2}\,dx$

$+ \int \frac{2}{(x+2)^2}\,dx$

$= \ln|x-1| - \ln|x+2| - \frac{2}{x+2}$

72a)



$A = \int_1^4 \frac{1}{x}\,dx = \ln x \Big|_1^4 = \underline{\underline{\ln 4}}$

$V = \pi \cdot \int_1^4 \frac{1}{x^2}\,dx = -\frac{\pi}{x}\Big|_1^4$

$= -\frac{\pi}{4} + 1 = \underline{\underline{\frac{3}{4}\pi}}$

Flächenschwerpunkt:

$2\pi y_s \cdot A = V \ \leadsto \ \underline{\underline{y_s = \frac{3}{8\cdot\ln 4}}}$

$A \cdot x_s = \int_1^4 x \cdot \frac{1}{x}\,dx = 3$

$\underline{\underline{x_s = \frac{3}{\ln 4}}}$

Volumenschwerpunkt:

$\underline{y_s = 0}$ auf Rot.-Achse

$x_s \cdot V = \int_1^4 \pi \cdot \left(\frac{1}{x}\right)^2 \cdot x\,dx$

$= \pi \cdot \ln x \Big|_1^4 = \pi \ln 4$

$x_s = \frac{\pi \cdot \ln 4}{\frac{3}{4}\pi} = \underline{\underline{\frac{4}{3}\ln 4}}$

b) $\int \left(x + \frac{1}{2}x^2\right)e^{-x}\,dx$

$= e^{-x}(-x-1) - \frac{1}{2}e^{-x}(x^2+2x+2)$

nach Tabelle

$= e^{-x}\left(-\frac{1}{2}x^2 - 2x - 2\right)$

$= -\frac{1}{2}e^{-x}(x^2+4x+4) = -\frac{1}{2}e^{-x}(x+2)^2$

2) $\int_{-2}^{1} f(x)\,dx = -\frac{1}{2}e^{-x}(x+2)^2\Big|_{-2}^{1}$

$= -\frac{9}{2e} = -\frac{9}{2}e^{-1}$

$\int_{-2}^{\infty} f(x)\,dx = \lim\limits_{t\to\infty} \int_{-2}^{t} f(x)\,dx$

$F(t) = -\frac{1}{2}e^{-x}(x+2)^2\Big|_{-2}^{t}$

$= -\frac{1}{2}e^{-t}(t+2)^2$

$\lim\limits_{t\to\infty} F(t) = 0$

3) Mittelwert in $[-2,0]$

$\bar{f} \cdot 2 = -\frac{1}{2}e^{-x}(x+2)^2\Big|_{-2}^{0}$

$= -\frac{1}{2} \cdot 4 = -2$

$\underline{\underline{\bar{f} = -1}}$

in $[-2,1]$:

$\bar{f} \cdot 3 = -\frac{9}{2}e^{-1}$

$\underline{\underline{\bar{f} = -\frac{3}{2}e^{-1}}}$

in $[-2,\infty]$ Mittelwert null.

c) $\int_0^a \left(\frac{1}{4}x^2 - 1\right)dx = \frac{1}{4}\frac{x^3}{3} - x\Big|_0^a$

$= \frac{a^3}{12} - a = a\left(\frac{a^2}{12} - 1\right) = 0$

$a = \sqrt{12}$

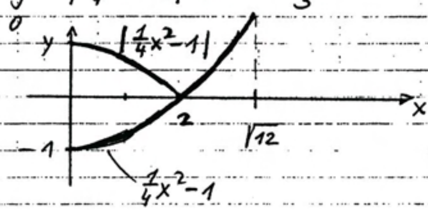
$\int_0^2 \left|\frac{1}{4}x^2 - 1\right|dx = \int_0^2 \left(1 - \frac{1}{4}x^2\right)dx$

$= x - \frac{x^3}{12}\Big|_0^2 = \frac{4}{3}$

Da $\int_0^{\sqrt{12}} \left(\frac{1}{4}x^2 - 1\right)dx = 0$

muß gelten

$\int_0^{\sqrt{12}} \left|\frac{1}{4}x^2 - 1\right|dx = \frac{8}{3}$



d) i) $W = \int_1^6 \left(\frac{4}{x^2} - \frac{4}{(x-10)^2}\right)dx$

$= -\frac{4}{x} + \frac{4}{x-10}\Big|_1^6$

$= \frac{25}{9}$

ii) $W = \int_1^9 \%\,dx$

$= -\frac{4}{x}\Big|_1^9 + \frac{4}{x-10}\Big|_1^6$

$= 0$