Lösungen Übungsblatt 8

66) i) $2\cdot\bar{f} = \int_0^2 \left(2-\frac{1}{2}x^2\right)dx$

$= 2x - \frac{1}{6}x^3\Big|_0^2 = \frac{8}{3}$

$\bar{f} = \frac{4}{3}$

$2\cdot f_{eff}^2 = \int_0^2 \left(2-\frac{1}{2}x^2\right)^2 dx$

$= \int_0^2 \left(4 - 2x^2 + \frac{1}{4}x^4\right)dx$

$= 4x - 2\frac{x^3}{3} + \frac{1}{20}x^5\Big|_0^2 = \frac{64}{15}$

$f_{eff} = \sqrt{\frac{32}{15}}$

ii) $\pi\cdot\bar{f} = \int_0^\pi \left(1-\frac{1}{2}\cos x\right)dx$

$= x - \frac{1}{2}\sin x\Big|_0^\pi = \pi$

$\bar{f} = 1$

$\pi f_{eff}^2 = \int_0^\pi \left(1-\frac{1}{2}\cos x\right)^2 dx$

$= x - \sin x + \frac{1}{4}\left(\frac{x}{2} + \frac{1}{2}\sin x\cos x\right)\Big|_0^\pi$

$= \pi + \frac{\pi}{8} = \frac{9}{8}\pi$

$f_{eff}^2 = \frac{9}{8}$; $f_{eff} = \frac{3}{2\sqrt{2}}$

iii) $\pi\cdot\bar{f} = \int_0^\pi (A\sin x + B\cos x)\,dx$

$= -A\cos x + B\sin x\Big|_0^\pi = 2A$

$\bar{f} = \frac{2}{\pi}\cdot A$

$\pi f_{eff}^2 = \int_0^\pi (A\sin x + B\cos x)^2 dx$

$= \int_0^\pi \left(A^2\sin^2 x + B^2\cos^2 x + 2AB\sin x\cos x\right)dx$

$= A^2\left(\frac{1}{2}x - \frac{1}{2}\sin x\cos x\right)\Big|_0^\pi$

$+ B^2\left(\frac{1}{2}x + \frac{1}{2}\sin x\cos x\right)\Big|_0^\pi$

$+ 2AB\cdot\frac{1}{2}\sin^2 x\Big|_0^\pi$

$= A^2\frac{1}{2}\pi + \frac{1}{2}\pi\cdot B^2 = \frac{1}{2}\pi(A^2+B^2)$

$f_{eff} = \sqrt{\frac{1}{2}(A^2+B^2)}$

67 a) $\int_1^t \frac{5+x}{x^3}dx = -\frac{5}{2}\frac{1}{x^2} - \frac{1}{x}\Big|_1^t$

$= -\frac{5}{2}\frac{1}{t^2} - \frac{1}{t} + \frac{5}{2} + 1 = F(t)$

$\lim_{t\to\infty} F(t) = \frac{7}{2}$

b) $\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx = 2\int_0^\infty \frac{1}{1+x^2}dx$

$\int_0^t \frac{1}{1+x^2}dx = \arctan x\Big|_0^t = \arctan t$

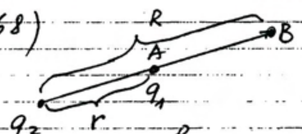
$2\int_0^\infty \frac{1}{1+x^2}dx = 2\lim_{t\to\infty}\arctan t = 2\cdot\frac{\pi}{2} = \pi$

c) $\int_0^t a e^{-ax}dx = a\cdot\frac{e^{-ax}}{-a}\Big|_0^t$

$= -e^{-at} + 1 = F(t)$

$\lim_{t\to\infty} F(t) = 1$

68)



$W_{AB} = \int_r^R \frac{q_1\cdot q_2}{x^2}\frac{1}{4\pi\varepsilon_0}dx$

$= \frac{1}{4\pi\varepsilon_0} q_1\cdot q_2\left(-\frac{1}{x}\right)\Big|_r^R$

$= \frac{1}{4\pi\varepsilon_0} q_1\cdot q_2\left(-\frac{1}{R} + \frac{1}{r}\right)$

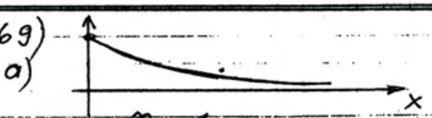
läßt man $B \longrightarrow \infty$ gehen ergibt sich:

$W = \frac{1}{4\pi\varepsilon_0} q_1\cdot q_2\,\frac{1}{r}$

Setzt man $q_1 = 1$ ergibt sich

$W = \frac{1}{4\pi\varepsilon_0} q_2\cdot\frac{1}{x}$

69) a)



$A = \int_0^\infty e^{-\frac{1}{2}x}dx = -2e^{-\frac{1}{2}x}\Big|_0^\infty$

$= -2e^{-\infty} + 2 = \underline{2}$

b) $V = \int_0^\infty \pi\left(e^{-\frac{1}{2}x}\right)^2 dx = \pi\int_0^\infty e^{-x}dx$

$= \pi\frac{e^{-x}}{-1}\Big|_0^\infty = \pi$

c) y_s kann mit Guldin berechnet werden

$2\pi\cdot y_s\cdot A = V$

$2\pi\cdot y_s\cdot 2 = \pi \sim \underline{y_s = \frac{1}{4}}$

$x_s = \frac{\int_0^\infty x e^{-\frac{1}{2}x}dx}{\int_0^\infty e^{-\frac{1}{2}x}dx}$

$= \frac{\left[\cancel{4}e^{-\frac{1}{2}x}\left(-\frac{1}{2}x - 1\right)\right]_0^\infty}{2}$

$= \frac{1}{2}\left(\cancel{4}e^0\cdot 1\right) = \underline{\underline{2 = x_s}}$

d) Schwerpunkt des Rot.-körpers

$y_s = 0$; Schwerp. auf Rot. Achse

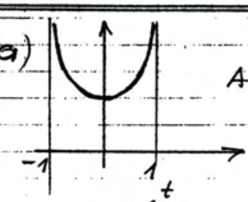
$V\cdot x_s = \int_0^\infty \pi\left(e^{-\frac{1}{2}x}\right)^2 x\,dx$

$= \int_0^\infty \pi\cdot x e^{-x}dx$

$= \pi\frac{e^{-x}}{1}(-x-1)\Big|_0^\infty = \pi$

$\underline{x_s = 1}$ vom Rot-körper

70) a)



$A = 2\cdot\int_0^1 \frac{1}{\sqrt{1-x^2}}dx$

$F(t) = 2\int_0^t \frac{1}{\sqrt{1-x^2}}dx = 2\cdot\arcsin x\Big|_0^t$

$= 2\arcsin t$

$\lim_{t\to 1} 2\arcsin t = 2\cdot\frac{\pi}{2} = \pi$

d.h. Flächeninhalt existiert

b) $V_x = 2\int_0^1 \pi\cdot\frac{1}{1-x^2}dx$

$F(t) = 2\int_0^t \pi\cdot\frac{1}{1-x^2}dx$

$= 2\pi\cdot\frac{1}{2}\ln\frac{1+x}{1-x}\Big|_0^t$

$= \pi\ln\frac{1+t}{1-t} - \pi\underbrace{\ln 1}_{0}$

$= \pi\left(\ln(1+t) - \ln(1-t)\right)$

$\lim_{t\to 1} F(t) = \pi\cdot\ln 2 - \pi\cdot\underbrace{\ln 0}_{\text{existiert nicht}}$

d.h. das Volumen ist unendlich groß; (existiert nicht)

c) $y_s\cdot 2\pi\cdot A = V$ Guldin Regel

d.h. auch y_s ist unendlich groß

Schwerpunkt liegt nicht im Endlichen

d.h. es gibt eine <u>endliche</u> Fläche, deren Schwerpunkt nicht endlich ist

71 a) $\int \sqrt{x}\left(x^2 + \frac{1}{x}\right)dx = \int\left(x^{5/2} + x^{-1/2}\right)dx$

$= \frac{2}{7}x^{7/2} + 2x^{1/2} = 2\sqrt{x}\left(1 + \frac{1}{7}x^3\right)$

b) $\int 2x\sqrt{1+4x^2}\,dx = \int \frac{1}{4}\sqrt{u}\,du$

$1+4x^2 = u$ $\qquad = \frac{1}{6}\sqrt{(1+4x^2)^3}$

c) $\int \frac{x+3}{(x+2)^4}dx = \int \frac{u+1}{u^4}du = \int\left(u^{-3} + u^{-4}\right)$

$= -\frac{1}{2}\frac{1}{u^2} - \frac{1}{3}\frac{1}{u^3}$

$= -\frac{1}{2}\frac{1}{(x+2)^2} - \frac{1}{2}\frac{1}{(x+2)^3}$

$= -\frac{1}{6}\frac{3x+8}{(x+2)^2}$

d) $\int \frac{2+5e^x}{3e^x}dx = \int\left(\frac{2}{3}e^{-x} + \frac{5}{3}\right)dx$

$= -\frac{2}{3}e^{-x} + \frac{5}{3}x$