Lösungen Übungsblatt 7

60) Mit Guldin – Regel

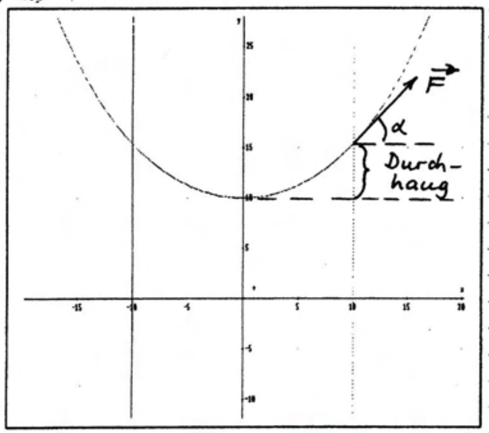
$x_s = 3$ wegen Symmetrie

$V_y = 2\pi \cdot x_s \cdot$ Fläche

$V_y = 2\pi \cdot 3 \left[\frac{1}{2} \cdot 2 \cdot 2 + 2 \cdot \frac{1}{2} \cdot 1 \cdot 1 \right]$

$V_y = 18\pi$

61) a)



Durchh. $D = y(10) - y(0)$

$D = 10(\cosh 1 - 1)$

$= 10 \cdot \frac{1}{2} \cdot \left[e^{1} + e^{-1} - 2 \right] = 5.43\,m$

$\tan\alpha = y'(10) = 10 \cdot \sinh 1 \cdot \frac{1}{10}$

$= \frac{1}{2}(e^{1} - e^{-1}) = 1.175$

$\alpha = 49.6^\circ$

b) $L = 2 \int_0^{10} \sqrt{1 + \sinh^2 \frac{x}{10}}\, dx$

es gilt: $\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x = 1 + \sinh^2 x$

$L = 2 \int_0^{10} \cosh \frac{x}{10}\, dx = 2 \cdot 10 \sinh \frac{x}{10} \Big|_0^{10}$

$L = 20 \cdot \sinh 1 = 10(e^{1} - e^{-1})$

$= 23.5\ m$

c) Kräfte in den Aufhängepunkten:

$m \cdot g$ = Kettenlänge $\cdot 7 \cdot 9.81\ N$

$F_{vertikal} = \frac{1}{2} mg = 806.87\ N$

$F_{horiz} = \cot\alpha \cdot F_{vert} = 687.70\ N$

62 a) Fläche $= ab - \frac{1}{4}\pi r^2 = A$

$V_x = \pi a \cdot b^2 - \frac{2}{3}\pi \cdot r^3$

$V_y = \pi a^2 b - \frac{2}{3}\pi r^3$

$x_s = \frac{V_y}{2\pi A} = \frac{a^2 b - \frac{2}{3} r^3}{2(ab - \frac{1}{4}\pi r^2)}$

$y_s = \frac{V_x}{2\pi \cdot A} = \frac{ab^2 - \frac{2}{3} r^3}{2(ab - \frac{1}{4}\pi r^2)}$

b) Fläche: $A = \frac{1}{2} 25a^2 - \frac{1}{2}a^2 - \pi a^2$

$A = a^2(12 - \pi)$

$V_y = \frac{1}{3}\pi (5a)^2 5a - \frac{1}{3}\pi a^3 - \frac{2}{3}\pi \cdot 8a^3$

$V_y = 36\pi a^3$

$V_x = \pi \cdot 4a^2 \cdot a + \frac{1}{3}\pi\, 4a^2 \cdot 4a - \frac{2}{3}\pi \cdot 8a^3$

$V_x = 32\pi a^3$

$y_s = \frac{V_x}{2\pi A} = \frac{16 a}{12 - \pi} \approx 1.806\, a$

$x_s = \frac{V_y}{2\pi \cdot A} = \frac{36\pi a^3}{a^2(12-\pi)} = \frac{18 a}{12 - \pi}$

$\approx 2.032\, a$

c) $x_s = 5$ bzw. 0 wegen Symmetrie

$A = 10 \cdot 2 + 2 \cdot 1 - \pi - 1 = 21 - \pi$

$V_x = \pi \cdot 2^2 \cdot 8 + \pi \cdot 3^2 \cdot 2 - 2 \cdot \frac{4}{3}\pi - 2\frac{1}{3}\pi$

$V_x = \frac{140}{3}\pi$

$y_s = \frac{V_x}{2\pi A} = \frac{140\pi}{3 \cdot 2\pi (21 - \pi)} \approx 1.307$

63) i) $\pi \cdot \bar{f} = \int_0^{\pi} (\sin x - 2\cos x)\, dx$

$\pi \cdot \bar{f} = (-\cos x - 2\sin x)\Big|_0^{\pi} = 2$

$\bar{f} = \frac{2}{\pi}$

ii) $\bar{g} \cdot 2 = \int_0^2 (1 + e^{-x})\, dx$

$\bar{g} \cdot 2 = (x - e^{-x})\Big|_0^2 = 3 - e^{-2}$

$\bar{g} = \frac{3 - e^{-2}}{2}$

b) $h_{eff}^2 \cdot 1 = \int_0^1 \left(1 - \frac{x^2}{2}\right)^2 dx$

$= \int_0^1 \left(1 - x^2 + \frac{x^4}{4}\right) dx$

$h_{eff}^2 = \left(x - \frac{x^3}{3} + \frac{x^5}{20}\right)\Big|_0^1 = \frac{43}{60}$

$h_{eff} = \sqrt{\frac{43}{60}}$

64 a) $\int \frac{e^x}{(1+e^x)^3}\, dx = \int \frac{1}{u^3}\, du$

$u = 1 + e^x \qquad = -\frac{1}{2} u^{-2}$

$\frac{du}{dx} = e^x \ ; \ dx = \frac{du}{e^x}$

$F(x) = -\frac{1}{2} \frac{1}{(1+e^x)^2}$

$\int \frac{1}{x \cdot (1 + \ln x)}\, dx = \int \frac{\frac{1}{x}}{1 + \ln x}\, dx$

$= \ln |1 + \ln x| = G(x)$

b) $3 \cdot h = F(3) - F(0)$

$3h = -\frac{1}{2} \frac{1}{(1+e^3)^2} + \frac{1}{8}$

$= \frac{(1+e^3)^2 - 4}{8(1+e^3)^2}$

$h = \frac{e^6 + 2e^3 - 3}{24(1+e^3)^2}$

$(e - 1) \cdot k = G(e) - G(1)$

$= \ln(1 + \ln e) - \underbrace{\ln(1 + \ln 1)}_{0}$

$= \ln 2$

$k = \frac{\ln 2}{e - 1}$

65) a) Man ermittelt den Mittelwert über eine volle Schwingung

$\omega = \frac{2\pi}{T} = 2\pi f$; f Frequenz

Mit Überlegung: $a_1 = 0$

mit Rechnung:

$a_1 \cdot T = -\frac{A}{\omega} \cos(\omega t + \varphi)\Big|_0^T$

$\cos(\omega T + \varphi) = \cos(2\pi + \varphi) = \cos\varphi$

$a_1 \cdot T = 0 \ \rightarrow\ a_1 = 0$

b) $\int_0^{T/2} A \sin(\omega t + \varphi)\, dt = -\frac{A}{\omega} \cos(\omega t + \varphi)\Big|_0^{T/2}$

$= -\frac{A}{\omega}\left[\cos(\pi + \varphi) - \cos\varphi\right] = 2\frac{A}{\omega}\cos\varphi$

$a_2 \frac{T}{2} = 2\frac{A}{\omega}\cos\varphi \ \rightarrow\ a_2 = \frac{2A\cos\varphi}{\pi}$

c) a_3 entspricht dem Sonderfall $\varphi = 0$ bei a_2: $\quad a_3 = \frac{2A}{\pi}$

d) $\int_0^T A^2 \sin^2(\omega t + \varphi)\, dt$

$= A^2 \left[\frac{1}{2} t - \frac{1}{2\omega} \sin(\omega t + \varphi)\cos(\omega t + \varphi) \right]_0^T$

$= A^2 \cdot \left[\frac{1}{2} T - \frac{1}{2\omega} \sin\varphi\cos\varphi + \frac{1}{2\omega} \sin\varphi\cos\varphi \right]$

$a_4^2 \cdot T = A^2 \cdot \frac{1}{2} T$

$a_4^2 = \frac{1}{2} A^2$

$a_4 = \frac{1}{\sqrt{2}} \cdot A$