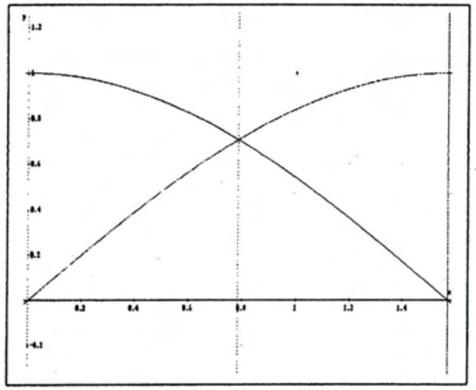
Lösungen Übungsblatt :

55 i)



i) $A = \int_0^{\pi/4} (\cos x - \sin x)\, dx$

$= \sin x + \cos x \Big|_0^{\frac{\pi}{4}}$

$= \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2} - 1 = \underline{\sqrt{2} - 1}$

ii) $A = \int_0^{\pi/4} (\cos x - \sin x)\, dx$

$+ \int_{\frac{\pi}{4}}^{\pi/2} (\sin x - \cos x)\, dx$

$= \sqrt{2} - 1 + \Big[-\cos x - \sin x\Big]_{\pi/4}^{\pi/2}$

$= \sqrt{2} - 1 + \Big[0 - 1 + \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}\Big]$

$= \underline{2(\sqrt{2} - 1)}$

dies hätte man auch mit Symmetrieüberlegungen erhalten.

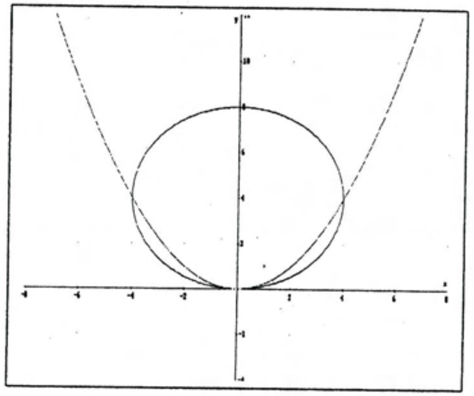
Man beachte:

$A \neq \int_0^{\pi/2} (\cos x - \sin x)\, dx$

$\int_0^{\pi/2} (\cos x - \sin x)\, dx = \sin x + \cos x \Big|_0^{\pi/2}$

$= 1 + 0 - 0 - 1 = 0$

56)



kleine Fläche:

Parabel oben – kreis unten

$A = 2 \int_0^4 \frac{1}{4} x^2\, dx - 2 \int_0^4 \left(4 - \sqrt{16 - x^2}\right) dx$

$= 2 \cdot \frac{x^3}{12}\Big|_0^4 - 2\Big[4x - \frac{1}{2} x \sqrt{16 - x^2} - 8 \arcsin \frac{x}{4}\Big]_0^4$

$= 2 \cdot \frac{16}{3} - 2\Big[16 - \frac{1}{2} \cdot 4 \cdot 0 - 8 \cdot \frac{\pi}{2}\Big]$

$= \frac{32}{3} - 32 + 8\pi = \underline{8\pi - \frac{64}{3}}$

große Fläche:

kreis oben – Parabel unten

$A = \pi \cdot 4^2 - \left(8\pi - \frac{64}{3}\right)$

$= \underline{8\pi + \frac{64}{3}}$

57) $\cosh x = \frac{1}{2}(e^x + e^{-x})$

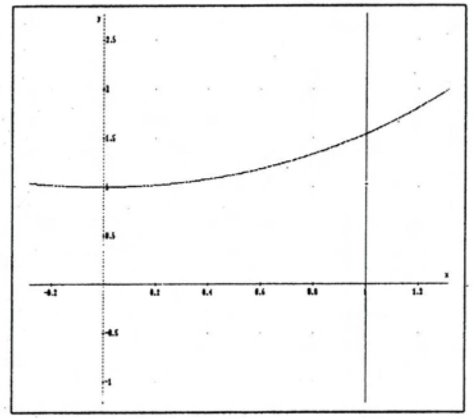
$V = \pi \int_0^1 \frac{1}{4} (e^x + e^{-x})^2\, dx$

$V = \pi \cdot \frac{1}{4} \int_0^1 (e^{2x} + 2 + e^{-2x})\, dx$

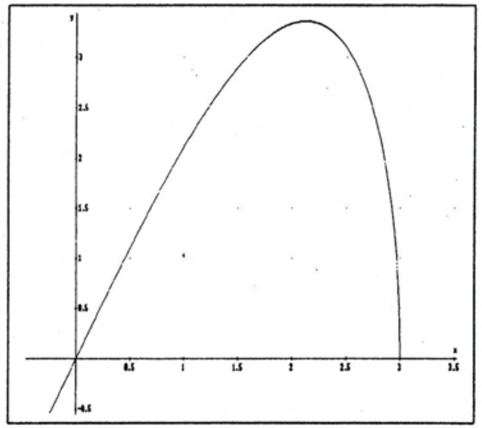
$V = \frac{1}{4}\pi \left(\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} \right)\Big|_0^1$

$= \frac{1}{4}\pi \left(\frac{1}{2}e^2 + 2 - \frac{1}{2}e^{-2} - \frac{1}{2} + \frac{1}{2} \right)$

$= \pi \left(\frac{1}{8}e^2 - \frac{1}{8}e^{-2} + \frac{1}{2} \right) \approx 4.417$



58)



$f(0) = f(3) = 0$

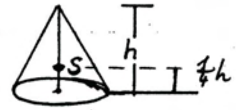
$V = \pi \int_0^3 f^2(x)\, dx = \pi \int_0^3 \frac{1}{9} x^2 \cdot 5 (9 - x^2)\, dx$

$= \frac{5}{9}\pi \int_0^3 (9x^2 - x^4)\, dx$

$= \frac{5}{9}\pi \Big[9\frac{x^3}{3} - \frac{x^5}{5} \Big]_0^3$

$= \frac{5}{9}\pi \Big[\frac{3^5}{3} - \frac{3^5}{5} \Big] = \underline{18\pi}$

59)



Der Schwerpunkt eines kreiskegels liegt $\frac{1}{4}h$ über der Grundfläche.

i) Mit Körperschwerpunkten

W_1: Arbeit, um das Wasser aus dem Kegel über den Rand zu pumpen.

$W_1 = \frac{1}{3}\pi \cdot 15^2 \cdot 10 \cdot \rho \cdot g \cdot \left(10 + \frac{10}{4}\right)$

$= 2.945 \cdot 10^4\, m^4 \cdot 9.81 \frac{m}{s^2} \cdot 10^3 \frac{kg}{m^3}$

$= 2.889 \cdot 10^8\, Nm$

W_2: Arbeit, um das Wasser aus dem kreiszylinder über den Rand zu pumpen.

$W_2 = \pi \cdot 15^2 \cdot 5 \cdot 7.5\, g \cdot \rho = 2.600 \cdot 10^8\, Nm$

$W_{ges} = W_1 + W_2 = 5.489 \cdot 10^8\, Nm$

ii) Mit Arbeitsintegralen

$W_1 = \int_0^{10} g \cdot \rho\, \pi \cdot x^2 (20 - y)\, dy$

$y = \frac{10}{15} x \rightarrow x = \frac{3}{2} y$

$W_1 = \pi g \rho \int_0^{10} \frac{9}{4} y^2 (20 - y)\, dy$

$= \pi \cdot g \rho \frac{9}{4} \left(\frac{20}{3} y^3 - \frac{y^4}{4} \right)\Big|_0^{10}$

$= 2.945 \cdot 10^4 \cdot g \cdot \rho = 2.889 \cdot 10^8\, Nm$

$W_2 = g \rho \int_0^5 \pi \cdot 15^2 \cdot (10 - y)\, dy$

$= \pi g \cdot \rho \cdot 15^2 \Big[10y - \frac{y^2}{2}\Big]_0^5$

$= 2.600 \cdot 10^8\, Nm$

$W_{ges} = W_1 + W_2 = 5{,}489 \cdot 10^8\, Nm$