Lösungen Übungsb. # 6

51c) $\int \frac{x+1}{x(x^2+1)}\,dx$

Partialbruchzerlegung

$$\frac{x+1}{x(x^2+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x} = \frac{Ax^2+Bx+Cx^2+C}{x(x^2+1)}$$

Koeff. vergleich:

$C=1 \quad B=1 \quad A=-1$

$$\int \%.\,dx = \int \frac{-x+1}{x^2+1}\,dx + \int \frac{1}{x}\,dx$$

$$= -\frac{1}{2}\int \frac{-2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx + \int \frac{1}{x}\,dx$$

$$= -\frac{1}{2}\ln(x^2+1) + \arctan x + \ln|x|$$

d) $\frac{2x+3}{(x-1)(x+1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$

$$A = \lim_{x\to 1} \frac{2x+3}{(x+1)(x+2)} = \frac{5}{6}$$

$$B = \lim_{x\to -1} \frac{2x+3}{(x-1)(x+2)} = \frac{1}{-2} = -\frac{1}{2}$$

$$C = \lim_{x\to -2} \frac{2x+3}{(x-1)(x+1)} = -\frac{1}{3}$$

$$\int \%.\,dx = \frac{5}{6}\int \frac{1}{x-1}\,dx - \frac{1}{2}\int \frac{1}{x+1}\,dx - \frac{1}{3}\int \frac{1}{x+2}\,dx$$

$$= \frac{5}{6}\ln|x-1| - \frac{1}{2}\ln|x+1| - \frac{1}{3}\ln|x+2|$$

d) $\frac{x^4-3x^2+x-1}{x^2-1} = x^2-2 + \frac{x-3}{x^2-1}$

(mit Polynomdivision)

$$\frac{x-3}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$A = \lim_{x\to -1} \frac{x-3}{x-1} = 2$$

$$B = \lim_{x\to 1} \frac{x-3}{x+1} = -1$$

$$\int \%.\,dx = \int (x^2-2)\,dx + \int \frac{2}{x+1}\,dx - \int \frac{1}{x-1}\,dx$$

$$= \frac{x^3}{3} - 2x + 2\ln|x+1| - \ln|x-1|$$

52) a) $|x^2-1| = |(x+1)(x-1)|$

$$= \begin{cases} x^2-1 & \text{für } x \geq 1 \\ 1-x^2 & \text{für } -1 \leq x \leq 1 \\ x^2-1 & \text{für } x \leq -1 \end{cases}$$

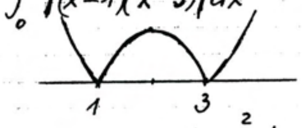
$$\int_0^2 |x^2-1|\,dx = \int_0^1 (1-x^2)\,dx + \int_1^2 (x^2-1)\,dx$$

$$= \left(x - \frac{x^3}{3}\right)\Big|_0^1 + \left(\frac{x^3}{3} - x\right)\Big|_1^2$$

$$= \frac{2}{3} + \frac{8}{3} - 2 - \frac{1}{3} + 1$$

$$= 2$$

b) $\int_0^4 |(x-1)(x-3)|\,dx$



$$|(x-1)(x-3)| = \begin{cases} x^2-4x+3 & x \geq 3 \\ -x^2+4x-3 & 1 \leq x \leq 3 \\ x^2-4x+3 & x \leq 1 \end{cases}$$

$$\int_0^4 \%.\,dx = \int_0^1 (x^2-4x+3)\,dx - \int_1^3 (x^2-4x+3)\,dx + \int_3^4 (x^2-4x+3)\,dx$$

$$= 4$$

c) $|x-1| + |x-3| = \begin{cases} -2x+4 & x \leq 1 \\ 2 & 1 \leq x \leq 3 \\ 2x-4 & x \geq 3 \end{cases}$

$$\int_0^4 \%.\,dx = \int_0^1 (-2x+4)\,dx + \int_1^3 2\,dx + \int_3^4 (2x-4)\,dx$$

$$= \left(-\frac{x^2}{2}\cdot 2 + 4x\right)\Big|_0^1 + 2x\Big|_1^3 + (x^2-4x)\Big|_3^4$$

$$= 10$$

e) $\int_0^{2\pi} |e^{-x}\sin x|\,dx$

$$= \int_0^{\pi} e^{-x}\cdot\sin x\,dx - \int_{\pi}^{2\pi} e^{-x}\sin x\,dx$$

$$\int e^{-x}\sin x\,dx = -\frac{1}{2}e^{-x}(\sin x + \cos x)$$

$$= -\frac{1}{2}e^{-x}(\sin x + \cos x)\Big|_0^{\pi} + \frac{1}{2}e^{-x}(\sin x + \cos x)\Big|_{\pi}^{2\pi}$$

$$= \frac{1}{2} + e^{-\pi} + \frac{1}{2}e^{-2\pi}$$

53 a) $\int \frac{\sin x}{1-\sin^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx$

$\cos x = u$; $\frac{du}{dx} = -\sin x$; $dx = \frac{-1}{\sin x}\,du$

$$= \int -\frac{1}{u^2}\,du = u^{-1} = \frac{1}{u} = \frac{1}{\cos x}$$

$$\int_0^{\pi/3} \%.\,dx = \frac{1}{\cos x}\Big|_0^{\pi/3} = 2-1 = 1$$

b) $\int \frac{\cos x - \cos^3 x}{1+\sin^3 x}\,dx = \int \frac{\cos x(1-\cos^2 x)}{1+\sin^3 x}\,dx$

$$= \int \frac{\cos x\cdot\sin^2 x}{1+\sin^3 x}\,dx = \underbrace{\frac{1}{3}\int \frac{3\sin^2 x\cos x}{1+\sin^3 x}}_{\frac{1}{3}\int \frac{f'}{f}dx}$$

$$= \frac{1}{3}\ln|1+\sin^3 x|$$

$$\int_0^{\pi/2} \%.\,dx = \frac{1}{3}\ln|1+\sin^3 x|\Big|_0^{\pi/2}$$

$$= \frac{1}{3}\ln 2$$

54) Fall $k \neq m$:

$$\int_0^{2\pi} \sin kx\cdot\sin mx\,dx$$

$$= \int_0^{2\pi} \frac{1}{2}\{\cos(k-m)x - \cos(k+m)x\}\,dx$$

$$= \frac{1}{2}\left[\frac{\sin(k-m)x}{k-m} - \frac{\sin(k+m)x}{k+m}\right]_0^{2\pi}$$

$$= 0$$

Fall $k = m$:

$$\int_0^{2\pi} \%.\,dx = \int_0^{2\pi} \frac{1}{2}\{\cos 0 - \cos 2kx\}\,dx$$

$$= \frac{1}{2}\left[x - \frac{\sin 2kx}{2k}\right]_0^{2\pi} = \frac{1}{2}\cdot 2\pi = \pi$$

$$\int_0^{2\pi} \sin kx\cdot\sin mx\,dx = \begin{cases} 0 & \text{für } k \neq m \\ \pi & \text{für } k = m \end{cases}$$

Fall $k \neq m$

$$\int_0^{2\pi} \cos kx\cdot\cos mx\,dx$$

$$= \int_0^{2\pi} \frac{1}{2}\{\cos(k-m)x + \cos(k+m)x\}\,dx$$

$$= \frac{1}{2}\left[\frac{\sin(k-m)x}{k-m} + \frac{\sin(k+m)x}{k+m}\right]_0^{2\pi} = 0$$

Fall $k = m$:

$$\int_0^{2\pi} \%.\,dx = \int_0^{2\pi} \frac{1}{2}\{\cos 0 + \cos 2kx\}\,dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2kx}{2k}\right]_0^{2\pi} = \pi$$

$$\int_0^{2\pi} \cos kx\cdot\cos mx\,dx = \begin{cases} 0 & \text{für } k \neq m \\ \pi & \text{für } k = m \end{cases}$$

$$\int_0^{2\pi} \sin kx\cdot\cos mx\,dx$$

$$= \int_0^{2\pi} \frac{1}{2}j(e^{-jkx} - e^{jkx})\frac{1}{2}(e^{jkx} + e^{-jkx})\,d$$

$$= \frac{1}{4}j\int_0^{2\pi} (e^{-j2kx} - e^{-j2kx})\,dx$$

$$= \frac{1}{4}j\left[\frac{e^{-j2kx}}{-j2k} + \frac{e^{-j2kx}}{j2k}\right]_0^{2\pi}$$

$= 0$ da die Werte an der oberen und an der unteren Grenze übereinstimmen.